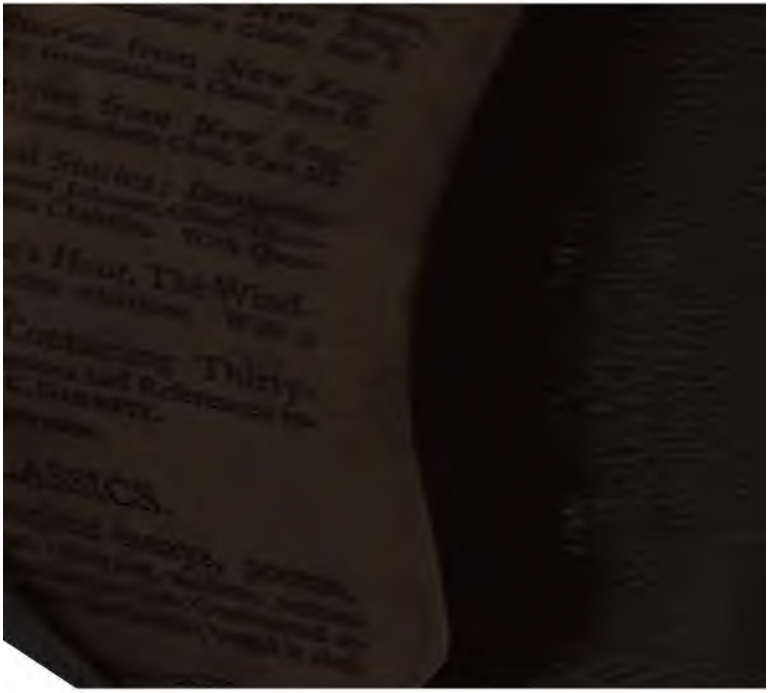

REVISED AND ENLARGED EDITION.

Warren Colburn's first Lessons

BOSTON:

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Warren Colburn's First Lessons.

INTELLECTUAL ARITHMETIC
UPON THE INDUCTIVE METHOD OF INSTRUCTION.

BY

WARREN COLBURN, A.M.

REVISED AND ENLARGED EDITION.

WITH AN

APPENDIX

CONTAINING A SKETCH OF THE AUTHOR'S LIFE, HIS ORIGINAL PREFACE, AND GEORGE B. EMERSON'S INTRODUCTION TO THE EDITION OF 1863.

BOSTON:
PUBLISHED FOR THE HEIRS OF WARREN COLBURN
BY HOUGHTON, MIFFLIN AND COMPANY.
The Riverside Press, Cambridge.
1884.

The Riverside Press, Cambridge:
Electrotyped and Printed by H. O. Houghton & Co.

PREFACE TO THE REVISED AND ENLARGED EDITION OF 1884.

In revising Warren Colburn's First Lessons in Intellectual Arithmetic, we have endeavored to make the *Colburn Method of Instruction* [1] more apparent and attractive.

The principal changes have been in the matter of Part I., Section I. (of the old edition), which has been augmented, and arranged in three sections.

In other portions of the book we have made changes in the arrangement, additions, and a few alterations. Those problems which related to prices and coins now out of date have been either changed or replaced by others.

In place of the Introduction to Written Arithmetic by Warren Colburn's son, which was contained in Part III., new matter has been substituted, and introduced into the present edition as Sections VI. and VII.

We trust that the simplicity of the first part of the book, and of the questions at the beginning of each section, will not

[1] The following extract from the Preface to Colburn's Algebra gives us, in Mr. Colburn's own words, a clear and simple description of his method of instruction: —

"In fact, explanations rather embarrass than aid the learner, because he is apt to trust too much to them, and neglect to employ his own powers; and because the explanation is frequently not made in the way that would naturally suggest itself to him, if he were left to examine the subject by himself. The best mode, therefore, seems to be, to give examples so simple as to require little or no explanation, and let the learner reason for himself, taking care to make them more difficult as he proceeds. This method, besides giving the learner confidence, by making him rely on his own powers, is much more interesting to him, because he seems to himself to be constantly making new discoveries. Indeed, an apt scholar will frequently make original explanations much more simple than would have been given by the author.

"This mode has also the advantage of exercising the learner in reasoning, instead of making him a listener, while the author reasons before him."

lead teachers to believe that all the questions are equally simple. We have tried so to grade the book that the pupil shall solve his own difficulties.

The Table of Contents, given on pages v to x, has been made full enough to show the plan of the book, and also to be of service in assigning lessons.

We believe that teachers will be much assisted in catching the spirit of the *Colburn Method* by carefully reading the Appendix to the present edition, in which will be found a short biographical sketch of Warren Colburn, the Preface to the first edition of his Arithmetic, and the Introduction, by George B. Emerson, to the edition of 1863.

We have prepared a box of counters and other aids, which will be found useful in lightening the work of the teacher, and in increasing the interest and power of the pupil. A description of these aids will be found on the inside of the last cover of this book.

THE EDITORS.

BOSTON, MASS., *March* 19, 1884.

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SECTION IX.

SECTION X.

SECTION XI.

SECTION XII.

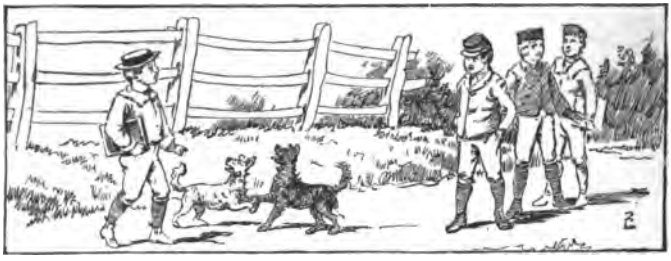
SECTION XIII.

APPENDIX.

A SHORT STORY

Which brings in the numbers from ONE *to* TEN, *inclusive.*

Harry is starting for school; his father's *two* dogs are frisking and barking before him, wishing that they could go too. A short distance off, where the two roads cross, *three* other boys are waiting for Harry to come along.



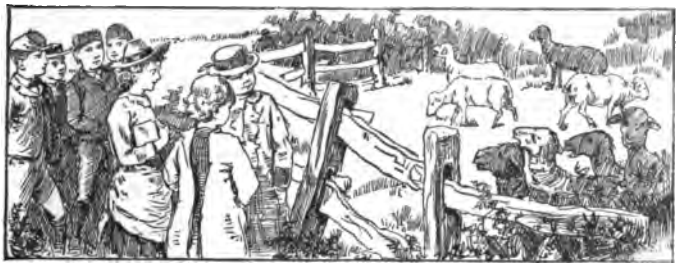
One Boy. Two Dogs. Three Boys.

The *four* boys have nearly reached farmer Brown's house, by the side of which they count one, two, three, four, *five* pretty trees; just beyond the house there are *six* hens eating.

Four Boys. Five Trees. Six Hens.

Three little girls have come from the house on the hill, and the *seven* children are talking so loudly that the sheep in the pasture near by have *run up to the bars* where the children sometimes

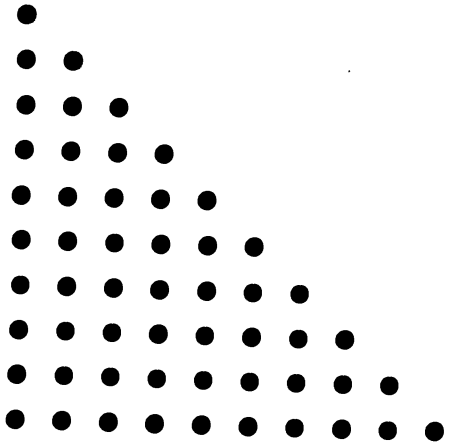
give them salt. Last year there was a large flock of sheep in the pasture, but now there are only *eight.*



Seven Children. **Eight Sheep.**

Farmer Brown is going to get a load of hay from his meadow, which is half a mile beyond the school-house; his two little girls, who are with him, have asked him to give the other children a ride; and now he and his load of *nine* children have nearly reached the school-house, where *ten* boys and girls are playing, while the teacher, with the school-bell in her hand, stands on the steps watching them.

Nine Children. **Ten Children.**


One.
Two.
Three.
Four.
Five.
Six.
Seven.
Eight
Nine.
Ten.

COLBURN'S ARITHMETIC.

SECTION I.

The First Ten Numbers.

A. Easy Problems, introducing the Numbers in Order.

1. How many heads have you? how many noses? how many mouths?

2. Point to one window; one book; one boy; one girl; one chair; one door; one table; one button.

3. How many hands have you? how many feet? how many eyes? how many ears?

4. John gives one apple to his sister and another to his cousin; how many apples does John give away in all?

5. If you take away one of two books, how many books will remain?

6. How many horses are there in a pair of horses?

7. If a slate pencil costs a cent, how much will two slate pencils cost?

8. How many letters are there in the word CAT? If you take away the A, how many letters will remain? If you put back the A and take away the C, how many letters will remain? If you take away the C and the A, how many letters will remain?

9. Walter had three cents, but he paid two cents for some candy; how many has he left?

10. If you have two nuts in one hand and one in the other, how many have you in both?

11. Elsie went to the post-office with a three-cent

piece to buy a postal card; how many cents should she bring back?

12. Susie had a present of a picture-book on her last birthday, and of a story-book last Christmas; two days ago her father gave her an arithmetic: how many books has Susie in all?

13. George found apples enough under a tree by the road-side to be able to give one to his father, one to his mother, and to keep one for himself; how many apples did he find?

14. How many two-cent postage-stamps can Walter Dale buy with a three-cent piece, and how much money will be left over? how many twos, then, are there in three? ●● ●

15. Not counting your thumb, how many fingers have you on your right hand? If you shut your forefinger and leave the rest open, how many will be open? If you shut your second finger also, how many will remain open? how many will be shut? If you shut your forefinger and your little finger, how many will remain open?

16. How many fore legs has a horse? how many hind legs? how many altogether?

17. John earned four cents by weeding, and spent one cent for a corn-ball, and two cents for a pipe to blow bubbles with; how much money has he left?

18. If you have two cents in one hand and two in the other, how many have you in both? If you have three cents in one hand and one in the other, how many have you in both?

19. From a heap of four counters take two away; take two more away: how many are there left? how

many pairs of counters, then, can you make out of four counters?

20. Lucy and her sister Mabel carried their lunch to school in the same basket: they found in the basket, among other good things, four apples; how many apples belonged to each?

21. If you count the fingers and the thumb on your right hand, how many will they make? If you count the fingers and the thumb on your left hand, how many will they make?

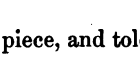
22. How many letters are there in TABLE? If you take away the T, how many will remain? If you take away the T and the B, how many will remain? If you put these back and take away the A, the L, and the E, how many will be left?

23. How many postal cards can you buy for five cents?

24. Sarah has two dolls and Clara has three; if Sarah gives her dolls to Clara, how many will Clara have? If Clara gives back two, how many will she have left?

25. James has two apples and Samuel three; how many apples have both together? If Samuel gives one of his apples to James, how many will James have then? how many will Samuel have left?

26. How many pairs of counters can you take out of a heap of five counters, by taking away a pair at a time? how many counters will there be left over?



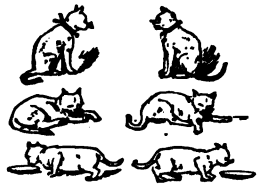
27. Olive's mother gave her a five-cent piece, and told her to go to the post-office and buy some two-cent stamps; how many stamps could Olive buy, and how much money would she have left over?

28. Joseph, Walter, and little Bob must carry into the cellar the wood which the dealer has dumped before their father's door. Joseph and Walter can carry two sticks at a time, but Bob can carry only one; how many sticks can they all carry at a trip?

29. If an orange costs three cents, how many oranges can Charlie buy with a five-cent piece, and how many cents will he have left over? how many threes, then, are there in five, and how many are left over?

Answer: ●●● ●● One three, two over.

30. If you count the thumb and fingers on one hand and one finger on the other, how many will there be?



31. Count these cats; how many are there? Count them by twos; how many twos are there? Count them by threes; how many threes are there?

32. George had three cents, and he earned three cents more by doing an errand for a neighbor; how many had he then? If he should spend five cents for some torpedoes, how many cents would he have left?

33. Helen found two pins on the school-room floor, and four more on her way home; how many did she find in all? Two of the pins were bent, and one was rusty; how many of the pins were good?

34. How many legs has a fly? how many are on the ground when he is using two to clean his wings with?

35. Berkeley and Alvaro found a horseshoe, which they sold to the blacksmith for six cents; how many cents belonged to each?

36. Berkeley, Alvaro, and John gathered a barrel of leaves, which they sold for six cents; how many cents belonged to each?

37. How many two-cent stamps can you buy for six cents? how many three-cent * stamps?

38. How many threes are there in six? how many twos?

●●●
●●●

39. How many fours are there in six?

One four, two over.
●●●● ●●

* No three-cent postage-stamps have been issued since Oct. 1, 1883.

40. How many fives are there in six? *Answer:* One five, ●●●●● one over. ●

41. Count these hens.

42. Walter has four cents and Joseph has three; how many have both together?

43. Robert gave five cents for an orange and two for an apple; how many cents did he give for both?

44. Grace picked up seven apples under a tree by the road-side; she ate one and gave the rest to her mother: how many did she give to her mother?

45. How many days are there in a week? Sunday is called the first day of the week; which is the third day? the fifth? the seventh? the second? the sixth?

Sunday.
Monday.
Tuesday.
Wednesday.
Thursday.
Friday.
Saturday.

46. How many days are there before Wednesday? how many after Wednesday?

47. There being no school on Saturday and Sunday, how many school-days are there in a week?

48. It takes a stage-driver two days to make a trip; how many trips can he make in a week, supposing, of course, that he rests on Sunday?

49. If a man can saw and split a cord of wood in three days, how many cords can he saw and split in a week?

50. Peter has seven cents, with which he wants to buy oranges; they cost three cents apiece: how many can he buy, and how much money will he have left?

51. How many two-cent stamps can John buy with seven cents, and how many cents will he have left?

52. How many twos are there in seven, and how many are left over? ●● ● ●● ●●

53. How many threes are there in seven, and how many are left over? ●●● ●
●●●

54. How many fingers have you on both hands? How many legs has a spider?

55. George gave five cents for a pint of peanuts, and three cents for an orange; how much did he give for both? How much more did he give for the nuts than for the orange?

56. Mrs. Smith has two arm-chairs and six other chairs in her parlor; how many chairs has she in all? Herbert and Edith once carried three of them into the sitting-room to play with; how many were left in the parlor?

57. Henry had seven blue marbles, and his father gave him one white one; how many had he then? how many more blue marbles had he than white ones?

58. Jennie had eight paper dolls, but she left two of them out in the rain, so that they were spoiled, and gave three away; how many had she left? Her mother then gave her four new ones; how many had Jennie then?

59. How many three-cent postage-stamps can you buy for eight cents, and how many cents will you have left over? ●●● ●●● ●●

60. David and Oliver carried a lady's valise to the railway station, and she gave them eight cents for their trouble; how many cents belonged to each?

61. A teamster, who was to be away from home all day, wished to take with him enough meal to give each of his four horses two quarts for dinner; how much meal did he take?

62. Bessie has knit eight stockings; how many pairs of stockings has she knit?

63. If you count the thumb and fingers on one hand and all the fingers on the other, how many will you have counted?

64. Arthur has some lead soldiers: eight are privates and one is an officer; how many has he in all?

65. Frank was sent to the grocer's to buy a box of mustard for seven cents and a lamp-wick for two cents; how much did he spend?

66. Emily's dress cost six dollars and her hat three dollars; how much more did the dress cost than the hat? how much did both together cost?

67. Dick had five plums, and John gave him four more; how many had Dick then?

68. Susan earned nine cents by hemming towels, and spent eight for a paper of needles; how much had she left?

69. If one orange cost three cents, how much must you pay for three oranges? ●●● ●●● ●●●

70. If there are three leaves on one stalk of clover, how many leaves will there be on three stalks?

71. Mr. Lanman paid for postage five cents on a letter to London, two cents on a postal card to Leipzig, and two cents on a letter to New York; how much did he pay in all?

72. When apples cost two cents apiece, how many can you buy for nine cents?

Answer: Four apples; and I shall have one cent left over.

73. Mrs. Eastman's guinea-hen laid seven eggs last week and two this week; how many did she lay in all?

74. How many fingers and thumbs have you on both hands?

75. Peter bought nine marbles, and afterwards found another in the street; how many marbles had he then?

76. How many one-cent postal cards can you buy for ten cents?

77. Bessie had ten chickens, but a hawk killed two of them; how many has she now?

78. Richard has a company of eight privates and two officers; how many more privates than officers has Richard? how many soldiers in all?

79. If you had six marbles in one hand and four in the other, how many more would you have in one hand than in the other? how many in both hands?

80. Emma found ten nuts under a tree, and generously gave seven to her little brother; how many had she left?

81. John and Joseph were coasting down hill on a sled which they had borrowed, when the rope caught under the runner and was broken: a new rope cost ten cents; how much ought each to pay?

82. How many two-cent stamps can you buy for ten cents?

B. Questions.

[The teacher will notice that the next sixty-five questions are arranged according to the *sums* of the numbers involved; thus in questions 4–8 the sum is *three;* in 9–14 the sum is *four.*

Let the pupils obtain the answers by the aid of counters; on the review the counters may be dispensed with.

Let the pupil make up examples for himself, applying the different numbers to objects; thus:—

"Three and two are five. John has three apples and I have two apples; both together we have five apples."]

1. One and one are how many?
2. How many ones are there in two?
3. How many must you add to one in order to get two?

4. One and two are how many?
5. Two and one are how many?
6. How many must you add to two to get three? how many to one?
7. How many ones are there in three?
8. How many twos are there in three?

Answer: One two and one over.

9. One and three are how many?
10. Two and two are how many?
11. Three and one are how many?
12. How many must you add to each of the following to get four: one? three? two?
13. How many twos are there in four?
14. How many threes are there in four?

Answer: One three and one over.

15. One and four are how many?
16. Two and three are how many?
17. Three and two are how many?
18. Four and one are how many?
19. How many must you add to each of the following to get five: one? three? two? four?
20. How many twos are there in five? how many threes? how many fours?

21. One and five are how many?
22. Two and four are how many?
23. Three and three are how many?
24. Four and two are how many?
25. Five and one are how many?
26. How many must you add to each of the following to get six: one? three? five? four? two?
27. How many twos are there in six? how many threes? how many fours? how many fives?

28. One and six are how many?
29. Two and five are how many?
30. Three and four are how many?
31. Four and three are how many?
32. Five and two are how many?
33. Six and one are how many?
34. How many must you add to each of the following to get seven: one? three? five? two? four? six?
35. How many twos are there in seven? how many threes? how many fours? how many fives? how many sixes?

36. One and seven are how many?
37. Two and six are how many?
38. Three and five are how many?
39. Four and four are how many?
40. Five and three are how many?
41. Six and two are how many?
42. Seven and one are how many?
43. How many must you add to each of the following to get eight: one? six? four? two? five? three? seven?
44. How many twos are there in eight? how many threes? how many fours? how many fives? how many sixes? how many sevens?

45. One and eight are how many?
46. Two and seven are how many?
47. Three and six are how many?
48. Four and five are how many?
49. Five and four are how many?
50. Six and three are how many?
51. Seven and two are how many?
52. Eight and one are how many?
53. How many must you add to each of the following to get nine: one? six? four? eight? three? seven? five? two?
54. How many twos are there in nine? how many threes? how many fours? how many fives? how many sixes? how many sevens? how many eights?

55. One and nine are how many?
56. Two and eight are how many?
57. Three and seven are how many?
58. Four and six are how many?
59. Five and five are how many?
60. Six and four are how many?
61. Seven and three are how many?
62. Eight and two are how many?
63. Nine and one are how many?
64. How many must you add to each of the follow-

ing to get ten: one? seven? five? three? eight? six? four? two? nine?

65. How many twos are there in ten? how many threes? how many fours? how many fives? how many sixes? how many sevens? how many eights? how many nines.

[The preceding questions should now be reviewed and answered without the aid of counters. The pupil should *never be told* an answer, however; he should find it out for himself, using counters or objects, if necessary.]

66. Harry Smith sells his pop-corn on the cars for five cents a bag. A bagful costs him two cents; how much does he make on each bag that he sells?

67. Harry sold three bags this morning while the train was waiting at Salem; what were his profits on these three?

68. George worked in the garden two hours on Monday, three hours on Tuesday, and four hours on Friday; how many hours did he work in all?

69. Aunt Elizabeth has given two cents apiece to Isabella, James, Adam, and Peter; how much has she given away in all?

70. Mrs. Pickett and Mary have been out of town to-day: they started at one o'clock, and did n't get back until ten o'clock; how many hours were they away?

71. Emily had ten peaches, but she generously gave away three to George, three to Sally, and three to Jenny; how many peaches had Emily left?

72. A quart of kerosene oil will fill Mrs. Swan's lamp four times: she has just had her two-quart can filled; how many times can she fill the lamp before all the oil in the can is used up?

73. Mr. Bell, the milkman, pays five cents a quart for his milk and sells it for seven; how much did Mr. Bell make on the five quarts which he sold to my mother this morning?

74. Henry has just spent a week at Plymouth: the railroad fares were two dollars, the board was six dollars, and other expenses were one dollar; how much did the visit cost?

75. One and one are how many?
76. Two and one are how many?
77. Three and one are how many?
78. Four and one are how many?
79. Five and one are how many?
80. Six and one are how many?
81. Seven and one are how many?
82. Eight and one are how many?
83. Nine and one are how many?

84. One and two are how many?
85. Two and two are how many?
86. Three and two are how many?
87. Four and two are how many?
88. Five and two are how many?
89. Six and two are how many?
90. Seven and two are how many?
91. Eight and two are how many?

92. One and three are how many?
93. Two and three are how many?
94. Three and three are how many?
95. Four and three are how many?
96. Five and three are how many?
97. Six and three are how many?
98. Seven and three are how many?

99. One and four are how many?
100. Two and four are how many?
101. Three and four are how many?
102. Four and four are how many?
103. Five and four are how many?
104. Six and four are how many?

105. One and five are how many?
106. Two and five are how many?
107. Three and five are how many?
108. Four and five are how many?
109. Five and five are how many?

110. One and six are how many?
111. Two and six are how many?
112. Three and six are how many?
113. Four and six are how many?

114. One and seven are how many?
115. Two and seven are how many?
116. Three and seven are how many?

117. One and eight are how many?
118. Two and eight are how many?

119. One and nine are how many?

120. One less one are how many?
121. Two less one are how many?
122. Three less one are how many?
123. Four less one are how many?
124. Five less one are how many?
125. Six less one are how many?
126. Seven less one are how many?
127. Eight less one are how many?
128. Nine less one are how many?
129. Ten less one are how many?

130. Two less two are how many?
131. Three less two are how many?
132. Four less two are how many?
133. Five less two are how many?
134. Six less two are how many?
135. Seven less two are how many?
136. Eight less two are how many?
137. Nine less two are how many?
138. Ten less two are how many?

139. Three less three are how many?
140. Four less three are how many?
141. Five less three are how many?

142. Six less three are how many?
143. Seven less three are how many?
144. Eight less three are how many?
145. Nine less three are how many?
146. Ten less three are how many?

147. Four less four are how many?
148. Five less four are how many?
149. Six less four are how many?
150. Seven less four are how many?
151. Eight less four are how many?
152. Nine less four are how many?
153. Ten less four are how many?

154. Five less five are how many?
155. Six less five are how many?
156. Seven less five are how many?
157. Eight less five are how many?
158. Nine less five are how many?
159. Ten less five are how many?

160. Six less six are how many?
161. Seven less six are how many?
162. Eight less six are how many?
163. Nine less six are how many?
164. Ten less six are how many?

165. Seven less seven are how many?
166. Eight less seven are how many?
167. Nine less seven are how many?
168. Ten less seven are how many?

169. Eight less eight are how many?
170. Nine less eight are how many?
171. Ten less eight are how many?

172. Nine less nine are how many?
173. Ten less nine are how many?

174. Five less two are how many?
175. Seven less three are how many?
176. Three less three are how many?
177. Nine less three are how many?
178. Six less two are how many?
179. Seven less four are how many?
180. Eight less three are how many?
181. Five less four are how many?
182. Seven less five are how many?
183. Nine less five are how many?
184. Eight less six are how many?

C. Miscellaneous Practical Problems.

1. Frank had nine pears, and gave two of them to Tom and three to Harry; how many had he left? how many more had he then than Harry?

2. Three boys, Peter, John, and Oliver, gave money to a beggar: Peter gave five cents, John three cents, and Oliver two cents; how much did the beggar receive?

3. A man bought a small firkin of butter for nine dollars: he used two dollars' worth, and sold the remainder for four dollars; how much did he lose?

4. Martha picked two quarts of berries for five cents a quart; how much did she earn? After she had spent two cents for ginger-snaps, how much had she?

5. A man bought a sheep for nine dollars, and to pay for it he gave five bushels of corn worth four dollars, and the rest in money; how much money did he pay?

6. A farmer brought to market five dollars' worth of potatoes and three dollars' worth of butter, and after he had sold these he bought a barrel of flour for six dollars; how much money did he carry home?

7. Samuel and John have a garden, in which they raise vegetables to sell: last year they raised two dollars' worth of early lettuce, three dollars' worth of tomatoes, and one dollar's worth of radishes; how much money did they make in all? what was the share of each?

8. Sarah was sent to the baker's shop with a ten-cent piece: she bought a loaf of bread for five cents and two cents' worth of yeast; how much in change did she receive?

9. Johnny had two cents this morning, but he earned five cents by holding a horse, and spent three cents for an orange; how much has he now?

10. Richard had three white marbles and five black ones: he lost two of them, and then bought four more; how many marbles had he then?

11. Lucy must knit ten times around her stocking every day: she knit eight rows this morning, but one had to be unraveled; how many rows must she knit this afternoon?

12. Jennie set her old hen on a nest of nine eggs: one of the eggs was bad, and the old hen broke two more; how many chickens were hatched out? Jennie gave two of her chickens away; how many had she left?

13. It is now Thursday, and Sarah has eight handkerchiefs to hem this week; but Clara says that she will do three of them in order that Sarah may have more time to play: if Sarah hems two handkerchiefs to-day and two to-morrow, how many will there be left for Saturday?

14. George earned nine cents by collecting three barrels of leaves: he spent three cents for candy, and gave two cents to his little brother; how much had he then?

15. James paid a carpenter five cents for two baskets of shavings, and then sold the shavings for four cents a basket; how much did he earn?

D. Numbers expressed by Figures.

Numbers are expressed not only by their names, but also, for the sake of brevity, by particular characters called figures.

Thus the number whose name is *one* is expressed also by the figure 1.

Names.	Figures
Nothing or zero	0
One	1
Two	2
Three	3
Four	4
Five	5
Six	6
Seven	7
Eight	8
Nine	9
Ten	10

NOTE. — The forms of figures used to express numbers, though always nearly the same, differ slightly in different styles of type.

[To the teacher : —

Before proceeding further the pupil should become perfectly familiar with the figures given above. He should be able both to know what they represent when he sees them and to write them neatly and distinctly. He should associate them in his mind with the numbers themselves rather than with the names of the numbers.

"First, show a number of objects, and then write (on the blackboard) the sign ; second, write the sign and ask the pupils to show that number of objects ; third, show a number of objects and have the pupils write the sign ; fourth, send the class to the board, then show numbers of objects one after the other, and have the pupils write the sign." — *Notes of Talks on Teaching given by F. W. Parker.*]

1. How many are 1 and 6? 5 and 4? 2 and 8? 3 and 4? 7 and 2? 2 and 5? 5 and 5? 4 and 6? 1 and 1? 3 and 5?

2. How many are 6 less 2? 9 less 3? 5 less 1? 7 less 6? 4 less 2? 8 less 4? 2 less 1? 3 less 2? 6 less 3? 9 less 4?

3. How many are 6 and 3?

In questions like this, 6 *and* 3 is often written, for short, 6+3. The sign + is called *plus*, and shows that the number after it is to be counted on or "added" to the number before it. Thus, 6+3 are 9 ; 2+8 are 10.

4. How many are 6+1? 8+2? 4+3? 2+7? 2+2?
3+2? 6+3? 7+1? 2+6? 2+4? 5+1?
4+4? 4+5? 3+7? 7+3? 1+8? 3+1?
5+2? 1+2? 4+1? 8+1? 6+2? 1+7?
4+2? 6+4? 5+3? 3+3? 3+6? 2+5?

5. How many are 6 less 3?

In questions like this, 6 *less* 3 is often written, for short, 6−3. The sign − is called *minus*, and shows that the number after it is to be taken away or "subtracted" from the number before it. Thus, 6−3 are 3; 8−4 are 4.

6. How many are 6−4? 9−2? 8−3? 4−1? 5−4?
7−5? 5−3? 5−2? 8−5? 8−6? 7−1?
6−5? 7−2? 7−3? 6−1? 9−8? 9−5?
9−7? 4−3? 9−6? 7−4? 8−7? 8−2?

7. How many are one and two and three? 3 and 3 and 2? 2+1+4? [that is, 2 and 1 and 4?] 4 and 2 and 2? 2+3+4? 6 and 2 and 1? 4+1+2? 2 and 1 and 5? 2+6+2? 1+2+7? three and one and three?

8. How many are 1+5 and 3? 7 and 1 and 2? one and one and eight? 6+2+1? 4+4+2? 3+2+4? 4+1+4? one and one and five? 3 and 3 and 3? 1 and 5+4? 2+3+3? 3+4+3? 7 and 1 and 1?

9. How many are 6 and 1 and 2? 8+1+1? 1 and 4 and 3? 3+2+3? five and two and one? 4+1+5? 2 and 2 and 1 and 3? 4 and 1 and 2 and 1? 3+1+2 and 4? 1+2+3+3?

10. How many are ten less two? 6 and 3 less 5? 6 and 2 less 3? 5+2−4? [that is, 5 and 2 less 4?]

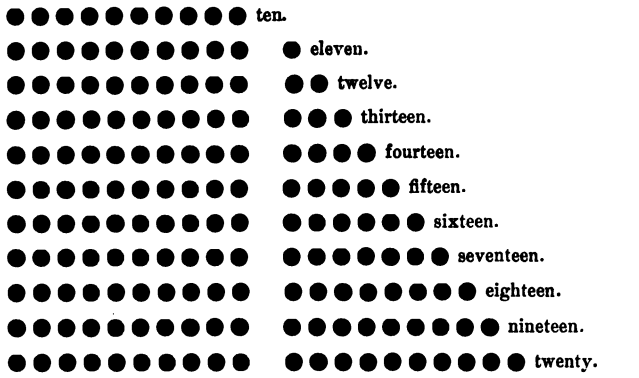
11. How many are 4−3+2? 5 and 3 less 6? seven less one less two? 6 and 2 less 4? 4+3−5? 8+1−2? 3+1−2? 6−5+2? 4+3 less 2?

12. How many are 8−3−2? [that is, 8 less 3 less 2?] 7 less 4 less 1? 4 and 4 less 2? 6+4−8? 5+3 less 7? 7 and 1 less 3? 2 and 5 less 4? 6+3−5? 4 and 2 less 5? 5+3−4?

SECTION II.

Numbers from Eleven to Twenty, inclusive.

A. The Names of the Numbers. Practical Problems introducing the Numbers in order.



1. Mr. White had 10 sheep, but he has just bought another one; how many has he now?

2. When Nettie rode in the omnibus, the other day, there were at first only 8 passengers, but 3 more soon got in; how many were there in all?

3. In Mrs. Hall's parlor are 6 pictures, and in her sitting-room there are 5; how many pictures are there in both rooms?

4. William has seven white marbles and four glass ones; Edward has sixteen: which has the more marbles, Edward or William?

5. A boy having eleven nuts gave away two of them; how many had he left?

6. If a man had eleven dollars, and should buy ten dollars' worth of coal, how much money would he have left?

7. Mary's hen laid 5 eggs week before last, and 7 eggs last week; how many in both weeks?

8. Two new pupils joined Ethel's Sabbath-school class last Sunday: before that there were ten in the class; how many are there now?

9. James received 8 cents for gathering bay leaves for Mr. Jones, the apothecary: James already had 4 cents of his own; how much did he have altogether?

10. A grocer bought a firkin of butter for nine dollars, and sold it at retail for three dollars more than he gave for it; how much did he sell it for?

11. Susan has 2 paper dolls, and each one of them has 6 dresses; how many dresses have they in all?

12. If you had 10 cents, and your papa should give you 3 more, how many would you have?

13. There are 8 plates and 5 saucers on the tea-table; how many plates and saucers?

14. If Flora reads four pages in her new story-book to-day, and nine to-morrow, how much will she have read altogether?

15. George caught 2 fishes last Monday: he sold one of them for 7 cents, and the other for 6 cents; how much did he get in all?

16. Albert has a case containing ten black lead pencils and four colored ones; how many pencils are there in the case?

17. Lucy was sent to buy 2 quarts of milk at 7 cents a quart; how much money did she spend?

18. There were fourteen pieces of kindling wood in the kitchen, but the servant used nine pieces in building the fire; how many were left?

19. In Miss Cole's school, 5 pupils sit in the first row, 5 in the second, and 4 in the third row; how many in all three rows?

20. Emily had fifteen raisins: she kept four, and gave the rest away; how many did she give away?

21. Mrs. Miller has 5 flower-pots in one window of her sitting-room, and 10 in the others; how many in all?

22. Sarah's teacher has twelve books on her table, and three in the table-drawer; how many books altogether?

23. Walter has two books: in one there are 7 pictures, in the other 8; how many are there in both books?

24. A man bought a sheep for 6 dollars, and a goat for 9 dollars; what did he give for both?

25. Fred paid ten cents for an ice-cream, and six cents for some cake to eat with it; how much did he spend in all?

26. A man bought a barrel of flour for eight dollars, and paid eight dollars more for a box of butter; how much did he pay in all?

27. Grace has two rose-bushes: on one there are nine roses, and on the other seven; how many roses has Grace?

28. The rag-man gave Anna sixteen cents for the rags that she had been saving; how much of this money had she left after spending ten cents for a needle-book?

29. Frank found ten pieces of flag-root, and Robert seven, when they were in the woods the other day; how many pieces of flag-root did both boys find?

30. Mr. Lane bought a newspaper for two cents, and some writing-paper for fifteen cents; how much did he spend in all?

31. A man who owed seventeen dollars paid all but six dollars; how much did he pay?

32. Alice sewed 9 hours yesterday, and 8 to-day; how many on both days?

33. There are seventeen songs in Kate's book, but she knows only four of them; how many has she yet to learn?

34. If the school-house has eight windows on one

side, and ten on the other, how many are there on both sides?

35. Horace bought for his mother, at the shop, six cents' worth of matches, and twelve cents' worth of spice; how much did he pay in all?

36. Martha has eleven white beads, and seven black ones; how many has she in all?

37. The butcher gave Tom and Lucy eighteen cents for the dandelions that they had dug; how much ought each to have?

38. Mr. Smith has two cows: one gives eleven quarts of milk a day, and the other seven; how many quarts of milk do both together give?

39. In Richard's class there are 10 boys and 9 girls; how many pupils are there in the class?

40. A man bought nineteen pounds of sugar, and, having lost a part through a hole in the paper, he found, when he reached home, that he had only fifteen pounds left; how much did he lose?

41. Mrs. Hill bought seven pounds of one kind of tea, and twelve pounds of another kind; how many pounds did she buy in all?

42. There are eleven mowers in the field, and eight boys to turn the grass; how many persons are in the field?

43. Emma sold two of her little chickens for ten cents apiece; how much did she receive?

44. Mr. Otis bought a barrel of flour for eight dollars, and a half-barrel of sugar for twelve dollars; how much money did he spend?

45. Clara uses six books at school, and has fourteen on her shelf at home; how many books has she?

46. Andrew broke a window in Mr. Smith's shop, and had to pay sixteen cents for a new pane of glass, and four cents for putty. Mr. Smith kindly offered to reset the glass for nothing; how much had Andrew to pay for his carelessness?

47. A lady went shopping one morning, and spent $4 * for a pair of shoes, and $10 for a bonnet; how much was that in all? If she had $20 in her pocket when she started out, how much ought she to have left?

48. If you can earn $2 in one week, how much can you earn in three weeks? If you work for four weeks, and spend $5 for a hat and a pair of boots, how much will you have left?

B. Numbers expressed by Figures. The Sign of Equality (=). Questions and Problems.

The figures corresponding to the numbers from ten to twenty, inclusive, are as follows: —

Names.		Figures.
Ten.		10
Eleven.	[10+1]	11
Twelve.	[10+2]	12
Thirteen.	[10+3]	13
Fourteen.	[10+4]	14
Fifteen.	[10+5]	15
Sixteen.	[10+6]	16
Seventeen.	[10+7]	17
Eighteen.	[10+8]	18
Nineteen.	[10+9]	19
Twenty.	[2 tens.]	20

1. How many are 6+5? 4+7? 3+8? 2+9? 10+1?

2. How many must you add to each of the following numbers to get 11: 1? 4? 3? 5? 8? 10? 9?

3. How many 5's in eleven?

4. How many are 1 and 11? 3+9? 2+10? 4+8? 7+5? 6+6?

5. How many must you add to each of the following numbers in order to get twelve: 5? 3? 8? 6? 2? 9?

6. How many 6's in 12? how many 2's? how many 4's? how many 3's?

* This is a short way of writing 4 dollars. Similarly $10 stands for 10 dollars.

7. How many are ten and three? 11+2? 5+8? 6+7? one and twelve? 9+4?

8. How many must you add to each of the following numbers in order to get 13: 2? 5? 3? 7? 9? 1?

9. How many 6's in thirteen? how many 2's?

10. How many are 2+12? nine and five? 7+7? 6+8? eleven and three? 10+4?

11. How many must you add to each of the following numbers in order to get fourteen: 3? 8? 4? 2? 9?

12. How many 7's in 14? how many 2's?

13. How many are fourteen and one? three and twelve? 8 and 7? 5 and 10? 9 and 6? eleven and four?

14. How many must you add to each of the following numbers in order to get 15: 3? 7? 9? 5? 1? 4?

15. How many 3's are there in 15? how many 5's?

16. How many are 8+8? 7+9? fourteen and two? 6 and 10? eleven and five?

17. How many must you add to 3 in order to get sixteen? how many to 8? to 5?

18. How many 8's are there in 16? how many 5's?

19. How many are 10 and 7? fourteen and three? 9+8? five and twelve? fifteen and two? four and thirteen? eleven and six?

20. How many must you add to each of the following numbers in order to get seventeen: 4? 6? 9? 3? fifteen? twelve?

21. How many 4's are there in 17?

22. How many are 10 and 8? 9 and 9? three and fifteen? five and thirteen? twelve and six? fourteen and four? seven and eleven?

23. How many must you add to each of the following numbers in order to get eighteen: 2? 7? 9? 4? 6? thirteen?

24. How many 9's are there in 18? how many 6's?

25. How many are 10 and 9? twelve and seven? three and sixteen? thirteen and six? four and fifteen?

26. How many must you add to each of the following numbers in order to get nineteen: 4? 3? 9? 6? thirteen? twelve?

27. How many 9's in 19?

28. How many are 10 and 10? three and seventeen? thirteen and seven? eight and twelve? four and sixteen? fifteen and five?

29. How many must you add to each of the following numbers in order to get twenty: 4? 3? 9? 14? 7? 5?

30. How many 10's are there in 20? how many 4's? how many 5's?

31. How many are 10+1? 11+1? 12+1? 13+1? 14+1? 15+1? 16+1? 17+1? 18+1? 19+1?

32. How many are 10+2? 11+2? 12+2? 13+2? 14+2? 15+2? 16+2? 17+2? 18+2?

33. How many are 10+3? 11+3? 12+3? 13+3? 14+3? 15+3? 16+3? 17+3?

34. How many are 10+4? 11+4? 12+4? 13+4? 14+4? 15+4? 16+4?

35. How many are 10+5? 11+5? 12+5? 13+5? 14+5? 15+5?

36. How many are 10+6? 11+6? 12+6? 13+6? 14+6?

37. How many are 10+7? 11+7? 12+7? 13+7?

38. How many are 10+8? 11+8? 12+8?

39. How many are 10+9? 11+9?

40. How many are 10+10?

41. How many are 15−3? 16−5? 19−2? 14−8? 20−13? 19−7? 17− 4? 13− 2? 18−6? 15−2? 18−12? 19−8? 16− 3? 15− 9? 17−6? 13−7? 20−12? 16−8? 14− 6? 12− 4? 20−8? 18−9? 14−11? 12−6? 17−11? 19−13?

Instead of writing the words "are" and "make" in such sentences as "6 and 3 are 9," or "6 and 3 make 9," we may use the sign of equality =. Thus 6+3=9 (which is to be read "six plus three equals nine") means that if we add three to six the result or SUM will be nine. Similarly 8−5 =3 ("eight less five equals three") means that if we subtract five from eight the result or DIFFERENCE will be three.

42. Read the following expressions:

8+9=17,	4+12=16,	7+ 8=15,
11+5=16,	11− 2= 9,	18−13= 5.

43. What number ought to be placed after the sign of equality in each of the following cases:

7+ 8=?	3+12=?	4+ 9=?
2+13=?	18− 5=?	17− 9=?
20− 7=?	19− 4=?	18− 7=?
19−11=?	14− 9=?	17−13=?
18− 3=?	13− 8=?	16− 7=?

C. Miscellaneous Problems.

1. Two boys, James and Robert, bought seven marbles apiece: James gave Robert four of his; how many had each then?

2. A man bought a firkin of butter for twelve dollars, but, it being damaged, he sold it again for eight dollars; how much did he lose?

3. A man bought three sheep for fifteen dollars, but could not sell them again for so much by eight dollars; how much did he sell them for?

4. A man bought sixteen pounds of coffee, and lost seven pounds of it as he was carrying it home; how much had he left?

5. A man, owing fifteen dollars, paid nine dollars of it; how much did he then owe?

6. John got fourteen pond lilies: he gave Helen two, and Lizzie two, and put the rest into a vase at home; how many were in the vase?

7. Farmer Higgins brought to market eight bushels

of potatoes worth six dollars, three bushels of onions worth four dollars, and a dollar's worth of cabbages: he sold them all; how much money did he receive?

8. How many are three and four and eight? nine and six and three? 4 and 9 and 2? 5 and 7 and 1? 2+8+6? 13 and 2 and 4? 3+7+9? 11+5+3? 14+1+2? 2+7+11? 8+2+9? 7+9+2? 2+3+4 and 5? 1+3+7+5? 2+4+6+8? What is the sum of 7, 9, and 3?

9. Samuel sold the old paper which he had collected for 19 cents: he bought a case of colored pencils for 15 cents; how much had he left?

10. John's mother gave him twenty cents, and sent him to buy six cents' worth of pepper and a ten-cent loaf of bread; how much ought John to have brought back from the shop?

11. If a barrel of flour is worth eight dollars, and a hundred-weight of sugar is worth twelve dollars, how much more is the sugar worth than the flour?

12. If a man had eleven dollars, and should buy some corn for five dollars, how much money would he have left?

13. A man bought a tub of butter for fifteen dollars, but it got slightly rancid during the hot weather, so that he was glad to sell it for eight dollars less than he gave for it; what did he sell it for?

14. How many are fifteen less four less two? 17 less 9 less 3? 11 less 2 less 4? 20 less 6 less 7? 19 less 8 less 5? 16—4—3? 18—1—5? 14—9—2? 13—5—3?

15. There are four rooms in Hattie's doll-house: there are three chairs in the kitchen, six in the dining-room, three in the bed-room, and five in the parlor; how many chairs has Hattie?

16. Henry and Arthur wish to buy a ball together; Henry has seven cents and Arthur six cents, but the ball costs sixteen cents; how many cents more must the boys earn before they can have the ball?

17. Sarah is to have eighteen days of vacation: she is to spend seven days at the seashore, and six more

with her aunt in the country; how many holidays will Sarah have at home?

18. How many are seven and five less two? 9 and 6 less 3? 8 and 7 less 9? 4+12—7? 6+11—4? 5+14 less 7? 8+2+4—3? 15—9+4? 18—7+2+1? 8+7 less 6+9?

19. A boy gave to one of his companions eight pears, to another six, to another four, and kept two himself; how many had he at first?

20. A boy found twenty apples under a tree by the road-side, and gave them to his school-mates as follows: to one he gave three, to another two, to another four, and to another five; how many did he give away? how many had he left?

21. John, George, and Robert fished, one day last summer, in the little brook near their uncle's house in the country: John caught six trout, George five, and Robert nine; but when they reached home they found that three of the fish had dropped off the twig on which the boys had strung them; how many trout were there left for the next morning's breakfast?

SECTION III.

Numbers above Twenty.

A. Names of the Numbers. Questions.

Names.		Figures.
Twenty.	[2 tens]	20
Twenty-one.	[20+1]	21
Twenty-two.	[20+2]	22
Twenty-three.	[20+3]	23
Twenty-four.	[20+4]	24
Twenty-five.	[20+5]	25
Twenty-six.	[20+6]	26
Twenty-seven.	[20+7]	27
Twenty-eight.	[20+8]	28
Twenty-nine.	[20+9]	29
Thirty.	[3 tens, or 20+10]	30
Thirty-one.	[30+1]	31
Thirty-two.	[30+2]	32
Forty.	[4 tens, or 30+10]	40
Fifty.	[5 tens, or 40+10]	50
Sixty.	[6 tens, or 50+10]	60
Seventy.	[7 tens, or 60+10]	70
Eighty.	[8 tens, or 70+10]	80
Ninety.	[9 tens, or 80+10]	90
One hundred.	[10 tens, or 90+10]	100

1. How many dots are there here?

●●●●●●●●●● ●●●●●●●●●● ●●●

2. Farmer Jones had 20 sheep, but he has bought 5 more; how many has he now?

3. Count these dots:

●●●●●●●●●● ●●●●●●●●●● ●

And these:

●●●●●●●●●● ●●●●●●●●●● ●●●●●●

And these:

●●●●●●●●●● ●●●●●●●●●● ●●●●●●●●●

4. Jenny paid at the store 2 cents for a lamp-wick, and 20 cents for a package of chocolate; how much did she pay in all?

5. There were 20 pupils in Miss Kilham's school last term, but this term there are 4 more; how many pupils are there now in the school?

6. Kate read 7 pages in her story-book to-day: she had already read 20 pages; how many pages has she read in all?

7. Henry sold 20 heads of lettuce out of his garden last week, and eight more this week; how many has he sold in all?

8. George earned 20 cents last week, and 10 cents more this week; how many cents has he earned in all?

9. James and Bertram have 20 marbles apiece, which they carry in the same bag; how many marbles are there in the bag?

10. Lawyer Brown has 30 books on the shelf above his table, and 10 more on the table itself; how many are there in all?

11. On Monday morning Ellen had 50 cents: she earned 10 cents on Monday by picking berries, and 10 cents more on Tuesday by doing some errands; how much money had she on Monday night? on Tuesday night?

12. On August 20th Farmer Tuttle had sold 70 bushels of early potatoes from the field next the barn: on the next Thursday he sent 10 bushels more to market; how many bushels did he sell in all?

13. Old Mr. Waite was 80 years old 10 years ago; how old is he now? how old will he be 10 years hence, if he lives so long?

14. What do we call 2 tens? *Answer:* Twenty. What do we call 3 tens? *Answer:* Thirty.

What do we call 4 tens? 5 tens? 6 tens? 7 tens? 8 tens? 9 tens? 10 tens? Express by figures the answers you have given.

15. What number is expressed by 50? 80? 20? 90? 100? 70? 30? 60? 40? 10? How many tens in each?

16. How is *thirty* expressed by figures? sixty? ninety? fifty? eighty? forty? one hundred? twenty? seventy?

17. 2 tens + 4 tens = how many tens? What is the answer called, and how is it expressed by figures?

Answer: 2 tens + 4 tens = 6 tens; six tens are called *sixty*, and are expressed by figures thus: 60.

How many, then, are 20 + 40? *Answer:* 60.

18. Answer in like manner the following: —

(1.) 1 ten + 8 tens = ? 10 + 80 = ?
(2.) 3 tens + 7 tens = ? 30 + 70 = ?
(3.) 6 tens + 2 tens = ? 60 + 20 = ?
(4.) 5 tens − 3 tens = ? 50 − 30 = ?
(5.) 8 tens − 4 tens = ? 80 − 40 = ?
(6.) 9 tens − 5 tens = ? 90 − 50 = ?
(7.) 2 tens + 1 ten + 4 tens = ? 20 + 10 + 40 = ?
(8.) 8 tens − 2 tens + 3 tens = ? 80 − 20 + 30 = ?
(9.) 6 tens + 4 tens − 5 tens = ? 60 + 40 − 50 = ?
(10.) 4 tens + 3 tens + 3 tens = ? 40 + 30 + 30 = ?

19. One ten is expressed by the figure 1 with a zero after it, thus: 10. Two tens are expressed by 2 with a zero after it, thus: 20; three tens by 3 with a zero after it, thus: 30. To express any number of tens, we write that number with a zero after it. Express by figures 18 tens (*Answer:* 180); 11 tens; 16 tens; 14 tens; 13 tens; 19 tens.

20. How many tens are there in 120? 100? 170? 150? 90?

21. *A* ONE *is sometimes called a* UNIT. Thus 6 stands for 6 *ones* or *units;* 7 for 7 ones or units; 12 for 12 ones or units; 10 for 10 ones or units. One ten, then, is the same as 10 units; 12, which is 10 + 2, is the same as 12 units, or one ten *and* two units. How many units in 16? in 12? 19? 15?

22. How many units in 20? 90? 80? 70? 50? 30? How many tens in each? *Answer*, to the first: In 20 there are twenty units or two tens.

23. How are 2 tens + 1 unit expressed by figures, and what is the result called?

Answer: 2 tens + 1 unit = 20 units + 1 unit = 21 units, and this is called *twenty-one.*

24. Give in like manner answers to the following: 2 tens + 2 units; 2 tens + 3 units; 2 tens + 4 units; 2 tens + 5 units; 2 tens + 6 units; 2 tens + 7 units; 2 tens + 8 units; 2 tens + 9 units; 3 tens + 7 units; 9 tens + 6 units; 8 tens + 5 units; 6 tens + 4 units; 7 tens + 7 units.

25. Name the numbers: 13, 23, 33, 43, 53, 63, 73, 83, 93; 14, 34, 54, 74, 94; 15, 25, 45, 65, 85, 95; 16 36, 66, 96; 17, 27, 57, 97; 18, 48, 68, 88; 19, 29, 39, 59, 79, 99.

26. Express by figures thirty-eight, ninety-seven, fifty-two, sixty-six, eighty-nine, thirty-two, forty-four, thirteen, twenty-two, ninety-nine, eighty-eight, seventy-seven, fifty-five, thirty-three, ninety-eight, seventy-eight, sixty-eight, fifty-eight, forty-eight, twenty-eight, eighteen, eighty-seven.

27. *A* DIME is a silver coin of the value of *ten cents.* 12 cents is equivalent to 1 dime + 2 cents. 24 cents is equivalent to 2 dimes + 4 cents.

What is the largest number of dimes you can get for 26 cents, and how many cents will you have left over? for 32 cents? for 98 cents? for 47 cents? for 50 cents? for 25 cents? for 75 cents? *Answer* to the first: For 26 cents you can get 2 dimes, and will have 6 cents left over.

28. What is the largest number of tens in 87, and how many units are left over? in 46? in 38? in 72? in 99? in 70? in 68? in 100?

Numbers above One Hundred.

Names.		Figures.
One hundred and one	[10 tens + 1, or 100 + 1]	101
One hundred and two	[10 tens + 2, or 100 + 2]	102
One hundred and three	[10 tens + 3, or 100 + 3]	103
One hundred and four	[10 tens + 4, or 100 + 4]	104
One hundred and five	[10 tens + 5, or 100 + 5]	105
One hundred and six	[10 tens + 6, or 100 + 6]	106

Names.		Figures
One hundred and seven	[10 tens + 7, or 100 + 7]	107
One hundred and eight	[10 tens + 8, or 100 + 8]	108
One hundred and nine	[10 tens + 9, or 100 + 9]	109
One hundred and ten	[10 tens + 1 ten, or 100 + 10]	110
One hundred and twenty	[10 tens + 2 tens, or 100 + 20]	120
One hundred and thirty	[100 + 30]	130
One hundred and forty	[100 + 40]	140
One hundred and fifty	[100 + 50]	150
One hundred and sixty	[100 + 60]	160
One hundred and seventy	[100 + 70]	170
One hundred and eighty	[100 + 80]	180
One hundred and ninety	[100 + 90]	190
Two hundred	[20 tens or 100 + 100]	200
Two hundred and one	[200 + 1]	201
Three hundred		300
Four hundred		400
Five hundred		500
Six hundred		600
Seven hundred		700
Eight hundred		800
Nine hundred		900
One thousand	[10 hundred]	1000

29. One hundred is expressed by the figure 1 with two zeros after it, thus: 100; two hundred is expressed by 2 with two zeros after it, thus: 200; three hundred by 3 with two zeros after it, thus: 300.

To express any number of hundreds we write that number with two zeros after it.

Express by figures 17 hundred. *Answer:* 1700.

Express by figures 14 hundred; 10 hundred; 8 hundred; 4 hundred; 26 hundred; 39 hundred.

30. How many hundreds are there in 600? 1600? 500? 900? 1100? 1500? 1000?

31. Name the numbers 800, 200, 1200, 400, 1900, 300, 100, 700. How many tens in each?

32. Express by figures the numbers three hundred,

seven hundred, six hundred, ten hundred, twenty hundred, thirteen hundred.

33. How are 1 hundred + 1 unit expressed by figures, and what is the result called?

Answer: 1 hundred (or 10 tens) + 1 unit = 100 units + 1 unit = 101 units, and this is called *One hundred and one.*

34. How many units are there in

1 hundred + 2 units?	1 hundred + 6 units?
1 hundred + 3 units?	1 hundred + 7 units?
1 hundred + 4 units?	1 hundred + 8 units?
1 hundred + 5 units?	1 hundred + 9 units?

Write your answers in figures.

35. How many are 1 hundred + 10 units?

Answer: 110.

36. How many are 1 hundred + 11 units?

Answer: 111.

37. How many are

1 hundred + 26 units?	6 hundred + 80 units?
4 hundred + 21 units?	7 hundred + 83 units?

38. How is one hundred and sixty-eight expressed by figures? two hundred and twenty-one? one hundred and twenty-seven? ten hundred and eleven? three hundred and one? five hundred and fifty-five?

39. Name the numbers 683, 769, 804, 240, 300, 227, 333, 999, 444, 555, 666, 784, 111, 888, 269, 624, 438.

B. Practical Problems.

1. Two boys went fishing together: one caught 16 fish, and the other 9; how many did they get in all?

2. Mr. Jones and Mr. Black pasture their cows in the same field: Mr. Jones has 14 cows, and Mr. Black has 12, how many cows are there together?

3. In Mrs. Hilton's parlor there are 3 windows, and each window has 8 panes of glass; how many panes are there in all these windows?

4. There are 5 pieces of sheet-music on the piano, and

18 more on the music-stand; how many pieces of music are there in all?

5. There are 6 books on the centre-table, and 16 more in the book-rack on the side-table; how many books are there in the room?

6. Miss Briggs has in her pen-box 18 fine steel pens, and 11 coarse ones; how many pens are there in the box?

7. In Mr. Thomas's garden there were 30 currant bushes, but 8 have been killed; how many still live?

8. A man bought an old sleigh for 17 dollars, and gave 9 dollars to have it repaired and painted, and then sold it for 23 dollars; how much did he lose by the bargain?

9. Johnny Howe's big brother, who is a clerk, has a salary of 20 dollars a month: last March he earned 8 dollars extra by evening work; how much did he earn in all in that month?

10. Hattie Brown, who lives near the railroad, counted one morning 13 trains going eastward, and 11 going westward; how many trains did she see in all?

11. When George was a small boy he saw 9 elephants in a menagerie, and this year he saw 16 more in the street procession when the circus came to town; how many elephants has George seen?

12. A boy, having 25 cents, bought one quart of cherries for 8 cents; one orange for 6 cents, and gave away 3 cents, how many cents had he left?

13. Mr. Jones, the druggist, pays 18 cents a box for tooth-powder and sells it for 26 cents a box; how much does he make on each box that he sells?

14. James earned 25 cents by shovelling snow, and 4 cents by doing an errand; how much did he earn in all?

15. Richard learned 17 verses of poetry last month, and he has learned 10 more this month; how many verses has Richard learned altogether?

16. A boy bought a box for 18 cents, and gave 8 cents to have it painted, and then sold it for 32 cents; how much did he gain by the bargain?

17. A boy gave some apples to his companions: to one he gave 7, to another 6, and to another 8; how many did he give away in all?

18. Samuel bought for his mother at the store a gallon of kerosene for 16 cents, and a loaf of bread for 5 cents; how much did Samuel spend? If his mother gave him twenty-five cents to pay for these articles, how much ought Samuel to bring home in change?

19. Mary has in her purse a ten-cent piece, two five-cent pieces, and a three-cent piece; how many cents does this make altogether?

20. A man bought a small firkin of butter for 9 dollars, a keg of molasses for 6 dollars, and 5 bushels of wheat for 7 dollars; how much did he give for the whole?

21. It is 12 miles by the railroad from Boston to Lynn, 4 miles from Lynn to Salem, and 12 miles from Salem to Ipswich; how many miles is it from Boston to Ipswich?

22. Farmer Jackson drove 10 miles to market this morning, but after he had sold his vegetables, he took the long road home, which made the distance 11 miles; how many miles has Farmer Jackson driven his horse to-day?

23. Mr. Trask paid 19 dollars for his boat, and 7 dollars for having her repaired and painted; how much did the boat cost in all?

24. In Mr. Green's orchard there are 3 rows of apple-trees, and there are 9 trees in each row; how many trees are there in all?

25. Mr. Wood paid 26 dollars for a suit of clothes, 3 dollars for a hat, and 5 dollars for a pair of boots; how much did the whole outfit cost?

26. Mr. Hurd bought 6 sheep for 28 dollars, and, after shearing them, sold the sheep for 25 dollars, and the wool for 6 dollars; how much did Mr. Hurd make by the bargain?

27. Johnny Foley bought 30 cents' worth of cakes of the baker, and sold them to some hungry carpenters,

who were building a house in the neighborhood, for 12 cents more than he gave for the cakes; how much did he sell them for?

28. Andrew spent 25 cents for his bat, and 20 cents for a ball; how much did both together cost?

29. Mr. Hall, the grocer, bought a box containing 30 bars of soap; how many bars had he left after he had sold 12 of them? Mr. Hall made 2 cents on each of these 12 bars; how much did he make in all?

30. Mr. White's silver watch cost him 36 dollars, and a chain 5 dollars; how much did he pay for both?

31. Mrs. Slade's new dining-table cost 28 dollars, and her chairs 20 dollars; how much did the whole cost?

32. There were 45 matches in the match-box, but 10 have been used; how many matches are there left?

33. Mary has read 56 pages in the book she has borrowed: there are 64 pages in the book; how many pages has Mary still to read?

34. Samuel earns 75 cents a week by carrying newspapers: he gives his mother 67 cents; how much does he keep for himself?

35. Charles's reading-book cost 48 cents, and his arithmetic 30 cents; how much did both together cost?

36. Mr. Perkins bought a lot of flour for 70 dollars, and sold it for 14 dollars more than he gave for it; for how much did he sell it?

37. Mr. Pike's sleigh cost 95 dollars, but he sold it for 87 dollars; how much did he lose?

38. January 1, 1884, was Tuesday; what day of the week was January 8? January 15? January 22? January 29? January 31? February 1?

39. The first Saturday in March, 1884, was March 1; what day of the month was the second Saturday? the third Saturday? the fourth Saturday? the fifth Saturday?

40. What day of the week is it? what day of the month? What day of the week was the first day of this month? on what day of the week will the last day of this month come? Date the Saturdays of this month.

[The teacher is advised to give many more problems like the last three, making them easy or hard, according to the abilities of the pupils.]

C. Questions.

1. How many are 9 and 2? 19 and 2? 29 and 2? 39 and 2? 49 and 2? 59 and 2? 69 and 2? 79 and 2? 89 and 2? 99 and 2?

2. How many are 9 and 3? 19 and 3? 29 and 3? 39 and 3? 49 and 3? 59 and 3? 69 and 3? 79 and 3? 89 and 3? 99 and 3?

3. How many are 9 and 4? 19 and 4? 29 and 4? 39 and 4? 49 and 4? 59 and 4? 69 and 4? 79 and 4? 89 and 4; 99 and 4?

4. How many are 9 and 5? 19 and 5? 29 and 5? 39 and 5? 49 and 5? 59 and 5? 69 and 5? 79 and 5? 89 and 5? 99 and 5?

5. How many are 9 and 6? 19 and 6? 29 and 6? 39 and 6? 49 and 6? 59 and 6? 69 and 6? 79 and 6? 89 and 6? 99 and 6?

6. How many are 9 and 7? 19 and 7? 29 and 7? 39 and 7? 49 and 7? 59 and 7? 69 and 7? 79 and 7? 89 and 7? 99 and 7?

7. How many are 9 and 8? 19 and 8? 29 and 8? 39 and 8? 49 and 8? 59 and 8? 69 and 8? 79 and 8? 89 and 8? 99 and 8?

8. How many are 9 and 9? 19 and 9? 29 and 9? 39 and 9? 49 and 9? 59 and 9? 69 and 9? 79 and 9? 89 and 9? 99 and 9?

9. How many are 9 and 10? 19 and 10? 29 and 10? 39 and 10? 49 and 10? 59 and 10? 69 and 10? 79 and 10? 89 and 10? 99 and 10?

10. How many are 8 and 3? 18 and 3? 28 and 3? 38 and 3? 48 and 3? 58 and 3? 68 and 3? 78 and 3? 88 and 3? 98 and 3?

11. How many are 8 and 4? 18 and 4? 28 and 4? 38 and 4? 48 and 4? 58 and 4? 68 and 4? 78 and 4? 88 and 4? 98 and 4?

12. How many are 8 and 5? 18 and 5? 28 and 5? 38 and 5? 48 and 5? 58 and 5? 68 and 5? 78 and 5? 88 and 5? 98 and 5?

13. How many are 8 and 6? 18 and 6? 28 and 6? 38 and 6? 48 and 6? 58 and 6? 68 and 6? 78 and 6? 88 and 6? 98 and 6?

14. How many are 8 and 7? 18 and 7? 28 and 7? 38 and 7? 48 and 7? 58 and 7? 68 and 7? 78 and 7? 88 and 7? 98 and 7?

15. How many are 8 and 8? 18 and 8? 28 and 8? 38 and 8? 48 and 8? 58 and 8? 68 and 8? 78 and 8? 88 and 8? 98 and 8?

16. How many are 8 and 9? 18 and 9? 28 and 9? 38 and 9? 48 and 9? 58 and 9? 68 and 9? 78 and 9? 88 and 9? 98 and 9?

17. How many are 7 and 4? 17 and 4? 27 and 4? 37 and 4? 47 and 4? 57 and 4? 67 and 4? 77 and 4? 87 and 4? 97 and 4?

18. How many are 7 and 5? 17 and 5? 27 and 5? 37 and 5? 47 and 5? 57 and 5? 67 and 5? 77 and 5? 87 and 5? 97 and 5?

19. How many are 7 and 6? 17 and 6? 27 and 6? 37 and 6? 47 and 6? 57 and 6? 67 and 6? 77 and 6? 87 and 6? 97 and 6?

20. How many are 7 and 7? 17 and 7? 27 and 7? 37 and 7? 47 and 7? 57 and 7? 67 and 7? 77 and 7? 87 and 7? 97 and 7?

21. How many are 7 and 8? 17 and 8? 27 and 8? 37 and 8? 47 and 8? 57 and 8? 67 and 8? 77 and 8? 87 and 8? 97 and 8?

22. How many are 7 and 9? 17 and 9? 27 and 9? 37 and 9? 47 and 9? 57 and 9? 67 and 9? 77 and 9? 87 and 9? 97 and 9?

23. How many are 6 and 5? 16 and 5? 26 and 5? 36 and 5? 46 and 5? 56 and 5? 66 and 5? 76 and 5? 86 and 5? 96 and 5?

24. How many are 6 and 6? 16 and 6? 26 and 6? 36 and 6? 46 and 6? 56 and 6? 66 and 6? 76 and 6? 86 and 6? 96 and 6?

25. How many are 6 and 7? 16 and 7? 26 and 7? 36 and 7? 46 and 7? 56 and 7? 66 and 7? 76 and 7? 86 and 7? 96 and 7?

26. How many are 6 and 8? 16 and 8? 26 and 8? 36 and 8? 46 and 8? 56 and 8? 66 and 8? 76 and 8? 86 and 8? 96 and 8?

27. How many are 6 and 9? 16 and 9? 26 and 9? 36 and 9? 46 and 9? 56 and 9? 66 and 9? 76 and 9? 86 and 9? 96 and 9?

28. How many are 5 and 6? 15 and 6? 25 and 6? 35 and 6? 45 and 6? 55 and 6? 65 and 6? 75 and 6? 85 and 6? 95 and 6?

29. How many are 5 and 7? 15 and 7? 25 and 7? 35 and 7? 45 and 7? 55 and 7? 65 and 7? 75 and 7? 85 and 7? 95 and 7?

30. How many are 5 and 8? 15 and 8? 25 and 8? 35 and 8? 45 and 8? 55 and 8? 65 and 8? 75 and 8? 85 and 8? 95 and 8?

31. How many are 5 and 9? 15 and 9? 25 and 9? 35 and 9? 45 and 9? 55 and 9? 65 and 9? 75 and 9? 85 and 9? 95 and 9?

32. How many are 4 and 7? 14 and 7? 24 and 7? 34 and 7? 44 and 7? 54 and 7? 64 and 7? 74 and 7? 84 and 7? 94 and 7?

33. How many are 4 and 8? 14 and 8? 24 and 8? 34 and 8? 44 and 8? 54 and 8? 64 and 8? 74 and 8? 84 and 8? 94 and 8?

34. How many are 4 and 9? 14 and 9? 24 and 9? 34 and 9? 44 and 9? 54 and 9? 64 and 9? 74 and 9? 84 and 9? 94 and 9?

35. How many are 3 and 8? 13 and 8? 23 and 8? 33 and 8? 43 and 8? 53 and 8? 63 and 8? 73 and 8? 83 and 8? 93 and 8?

36. How many are 3 and 9? 13 and 9? 23 and 9? 33 and 9? 43 and 9? 53 and 9? 63 and 9? 73 and 9? 83 and 9? 93 and 9?

37. How many are 2 and 9? 12 and 9? 22 and 9? 32 and 9? 42 and 9? 52 and 9? 62 and 9? 72 and 9? 82 and 9? 92 and 9?

DRILL TABLES IN SUBTRACTION.

A.	B.	C.	D.	E.	F.	G.	H.	I.
19	18	17	16	15	14	13	12	11
29	28	27	26	25	24	23	22	21
39	38	37	36	35	34	33	32	31
49	48	47	46	45	44	43	42	41
59	58	57	56	55	54	53	52	51
69	68	67	66	65	64	63	62	61
79	78	77	76	75	74	73	72	71
89	88	87	86	85	84	83	82	81
99	98	97	96	95	94	93	92	91

38. Take 2 from each number in A; also 3, 4, 5, 6, 7, 8, 9, 10.

[The teacher may read this question to the pupils as follows: How many are 19 less 2 ? 29 less 2 ? etc., 19 less 3 ? 29 less 3 ? etc. And in like manner the following questions may be read.]

39. Take 2 from each number in B; also 3, 4, 5, 6, 7, 8, 9, 10.

40. Take 2 from each number in C; also 3, 4, 5, 6, 7, 8, 9, 10.

41. Take 2 from each number in D; also 3, 4, 5, 6, 7, 8, 9, 10.

42. Take 2 from each number in E; also 3, 4, 5, 6, 7, 8, 9, 10.

43. Take 2 from each number in F; also 3, 4, 5, 6, 7, 8, 9, 10.

44. Take 2 from each number in G; also 3, 4, 5, 6, 7, 8, 9, 10.

45. Take 2 from each number in H; also 3, 4, 5, 6, 7, 8, 9, 10.

46. Take 2 from each number in I; also 3, 4, 5, 6, 7, 8, 9, 10.

D. Miscellaneous Problems and Questions.

1. Mary's aunt gave her fifty cents; how much had Mary left after she had paid thirty-five cents for a tea-set for her doll-house?

2. Emma's patch-work quilt is to contain forty-nine squares when it is finished: she has already made thirty-two squares; how many squares has she still to make?

3. It takes Walter twenty minutes to walk to school, but at noon, when he is hungry, he generally walks home in fifteen minutes; how many minutes does Walter spend on his way to and from school?

4. William bought some apples for sixty-two cents, and sold them one by one for eighty-four cents; how much did he make by the operation?

5. How many are fourteen and eight and seven? twenty-four and eight and seven? seventy-four and eight and seven? 32 and 9 and 3? 42+9+3? 62+9+3? 17+3+8? 37+3+8? 41+12+4? 61+12+4?

6. Mr. Hill paid thirty cents for a bottle of ink, fifteen cents for blotting paper, and twelve cents for steel pens; how much did the whole cost?

7. If Anna goes to the grocer's with sixty-five cents, and buys there a pound of butter for thirty-five cents, and a loaf of bread for ten cents; how much money ought she to bring home?

8. How many are 32 and 7 less 9? 42 and 7 less 9? 38 and 5 less 7? 64 and 9 less 8? 38 and 6 and 4 less 7? 52 and 6 and 4 and 5? 67 and 3 and 4 and 5?

9. Mr. Rand, the printer, charged Miss Beckford sixty cents for a pack of visiting cards; he had to pay eighteen cents for the blank cards; how much did he receive for the trouble of printing on them?

10. Henry paid sixty-two cents for a fishing-pole, twenty cents for a line, and eight cents for some hooks, how much did the whole cost?

11. How many are 57 and 6 and 3 and 5 and 2 less 8? 83 and 5 and 6 and 2 less 4 less 7? 75 and 8 and 9 and 3 less 7 less 6?

12. What number ought to be placed after the sign of equality in each of the following cases: —

$$43+12+8-4+6=?$$
$$38+\ 8+6-5-2=?$$
$$82-10-7-3+5=?$$

13. Ernest broke a window, and it cost eighty cents to reset it: he had one dollar of his own; how much had he left after paying for the window?

14. Mr. Ames paid eighteen dollars for gas last year, and seventy-six dollars for coal; how much did he pay for both?

15. How many are

$$78+7+6+2+5+8-9?$$
$$84+2+9-7-8+3+4?$$
$$37+4+5+4-8-7-9?$$

16. A man bought a cow for twenty-eight dollars, a calf for twelve dollars, a sheep for six dollars, and a pig for seven dollars; how much did he give for the whole?

17. James had 27 cents: John gave him 4 more, David 7, and George 11; how many had James then?

18. A man paid 16 dollars to A, 9 dollars to B, 7 dollars to C, 10 dollars to D, 6 dollars to E, 4 dollars to F, and had 8 dollars left; how many had he at first?

19. Mr. Gray paid eighty dollars for his passage from New York to Liverpool: the railway fare from his home to New York was fourteen dollars, and carriage fare in New York two dollars; how much did Mr. Gray pay for his fare from his home to Liverpool?

20. Mrs. Davis took her children, George and Lucy, to the concert: horse car fares cost thirty cents, and the tickets eighty cents; how much did the whole cost?

21. Martha carried a dollar-bill to the shop, and bought there a pound of tea for seventy cents, and two pounds of sugar for twenty-two cents; how much change did she receive?

22. Mrs. Wood paid ninety cents for some rose bushes, and twenty-eight cents for some violet roots to plant in her garden; how much did they all cost?

23. From Boston to Roxbury it is three miles; from

Roxbury to Dedham, six miles; from Dedhan to Walpole, eleven miles; from Walpole to Wrentham, four miles; from Wrentham to Attleborough, four miles; from Attleborough to Pawtucket, nine miles; from Pawtucket to Providence, four miles; how many miles is it from Boston to Providence?

24. One boy had fifteen nuts: another boy gave him seven; another, nine; and another gave him enough to make his number forty; how many did the last boy give him?

25. A drover bought sheep as follows: of one man he bought twenty-seven, of another eight, of another ten, and of another five: afterwards he sold nine of them; how many had he then?

26. A lady bought a comb for thirty-seven cents, some tape for eight cents, some pins for ten cents, some needles for six cents, and some thread for six cents: she gave the salesman seventy-five cents; how much change ought she to receive back?

27. A man owed fifty-six dollars: at one time he paid seventeen dollars; at another, eight; at another, five; at another, seven; at last he paid the rest of the debt, wanting four dollars; how much was the last payment?

28. Six men bought a horse for one hundred and twenty dollars: the first gave twenty-three dollars; the second, fifteen; the third, twelve; the fourth, nine; the fifth, seven; how much did the sixth give?

29. A man bought a horse for one hundred dollars, and paid fifteen dollars for keeping him; he let him enough to receive twenty dollars; and then sold him for forty-three dollars; did he gain or lose by the bargain? and how much?

30. Sarah's new book cost one dollar and twenty cents, and her pencils sixteen cents; how much did both together cost?

31. A man bought a horse for one hundred and forty dollars, and sold him for twelve dollars less than he gave for him; for how much did the man sell his horse?

SECTION IV.

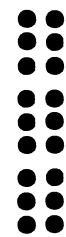
Multiplication.

A. Problems bringing in, in order, the Numbers below Thirteen.

ONE.

1. At a cent a stick, how much will six sticks of candy cost? How many are twice one? five times one? seven times one? nine times one? twelve times one?

TWO.



2. If apples cost two cents apiece, what will two apples cost?

3. If one yard of tape costs two cents, how much will three yards cost?

4. How much must you pay for four two-cent postage-stamps?

5. George was sent to buy six skeins of black flax thread; if each skein costs two cents, how much had George to pay for the whole?

6. The baker left ten two-cent rolls at Mrs. Smith's house this morning; how much had she to pay for them all?

7. There are above two dots in each row. If you count together two rows, how many dots will you have counted? three rows? five rows? How many, then, are once two? twice two? three times two? four times two? five times two? six times two? seven times two? eight times two? nine times two? ten times two? eleven times two? twelve times two?

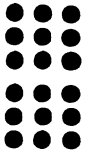
THREE.

8. If peaches are three cents apiece, how much will four cost?

9. If there are three feet in one yard, how many feet are there in five yards?

10. Each stalk of clover has generally three leaves; how many leaves would there be on seven clover-stalks?

11. If a barrel of Baldwin apples is worth three dollars, how many dollars are nine barrels worth?



12. In the Ashton Town hall there are ten gas fixtures, and each fixture has three gas jets; how many jets are there in the hall?

13. If oranges are worth three cents apiece, how much will a dozen cost?

14. In the figure printed above there are three dots in each row. If you count together three rows, how many will there be? six rows? eight rows? How many are once three? twice three? three times three? four times three? five times three? six times three? seven times three? eight times three? nine times three? ten times three? eleven times three? twelve times three?

FOUR.

[The following problems may be illustrated by rows of dots or counters, as above.]

15. If a man can walk four miles in one hour, how far can he walk in three hours?

16. If cloth costs four dollars a yard, how much will six yards cost?

17. If there are four quarts in one gallon, how many quarts are there in five gallons?

18. There are four pecks in a bushel; how many pecks of corn are there in eight bushels?

19. There are twelve doors in this house, and each door has four panels; how many panels have they altogether?

20. How many are once four? twice four? three times four? four times four? five times four? six times four? seven times four? eight times four? nine times four? ten times four? eleven times four? twelve times four?

FIVE.

21. If a baker's loaf of bread costs five cents, what will two loaves cost? four loaves?

22. If Mary knits five rows a day, how many rows will she knit in three days? six days?

23. John can carry five pieces of fire-wood at a time from the cellar to the kitchen; how many pieces can he carry up in five trips?

24. How much are eight five-dollar bills worth?

25. How much are twelve five-cent nickels worth?

26. How many are once five? twice 5? 3 times 5? 4 times 5? 5 times 5? 6 times 5? 7 times 5? 8 times 5? 9 times 5? 10 times 5? 11 times 5? 12 times 5?

SIX.

27. If a man works six days in each week, how many days will he work in the course of three weeks?

28. If a horse can go six miles an hour on the road, how far can he go in five hours?

29. A woman earns six dollars a week; how much does she earn in eight weeks?

30. Mr. Hall pays six cents a quart for picking and cleaning his currants. John picked ten quarts and Sarah eleven; how much did each earn?

31. In Miss Prince's school-room there are six seats in each row, and there are five rows of seats; how many seats are there in the room?

32. How many are once six? twice six? 3 times 6? 4 times 6? 5 times 6? 6 times 6? 7 times 6? 8 times 6? 9 times 6? 10 times 6? 11 times 6? 12 times 6?

SEVEN.

33. There are seven days in one week; how many days are there in three weeks?

34. If one quart of milk costs seven cents, how many cents will four quarts cost? six quarts?

35. Miss Blake paid seven dollars a week last summer for board at the seaside, and she stayed five weeks; how much was her board bill?

36. Mr. Cobb, the grocer, puts up his oatmeal in seven-pound bags; how many pounds of meal does it take to fill ten bags?

37. Mr. Hall has eight rows of raspberry bushes, and there are seven bushes in each row; how many bushes are there altogether?

38. How many are once seven? twice seven? 3 times 7? 4 times 7? 5 times 7? 6 times 7? 7 times 7? 8 times 7? 9 times 7? 10 times 7? 11 times 7? 12 times 7?

EIGHT.

39. There are in this room three windows, and eight panes in each window; how many panes of glass are there in the room?

40. Mr. Otis took his four children into the apothecary's and gave each a glass of soda-water; how much did the four glasses cost at eight cents a glass?

41. If there are eight quarts in one peck, how many quarts are there in five pecks? in seven pecks?

42. If a barrel of flour costs eight dollars, what is the cost of six barrels?

43. If it takes a man eight hours to saw a cord of wood, how many working hours will it take to saw eight cords? ten cords?

44. How many are once eight? twice eight? 3 times 8? 4 times 8? 5 times 8? 6 times 8? 7 times 8? 8 times 8? 9 times 8? 10 times 8? 11 times 8? 12 times 8?

NINE.

45. There are nine windows in each story of Mr. Smith's house; how many windows are there in the two upper stories? in all three stories?

46. If a package of envelopes cost nine cents, how much will four packages cost?

47. A workman who has no hod carries bricks to the top of a house in his arm; if he can carry nine bricks at each trip, how many can he carry in eight trips?

48. A certain gas light consumes nine feet of gas per hour; how much will it burn in six hours?

49. How many are once nine? twice nine? 3 times 9? 4 times 9? 5 times 9? 6 times 9? 7 times 9? 8 times 9? 9 times 9? 10 times 9? 11 times 9? 12 times 9?

TEN.

50. If note-paper costs ten cents a quire, what will two quires cost? three quires? five quires?

51. If a cake of soap sells for ten cents, how much will a grocer receive by selling at retail a box of eight cakes?

52. How many cents are six ten-cent pieces worth? ten ten-cent pieces?

53. Mr. Knapp pays ten dollars a month rent; how much does he pay in a year?

54. How many are once ten? twice ten? 3 times 10? 4 times 10? 5 times 10? 6 times 10? 7 times 10? 8 times 10? 9 times 10? 10 times 10? 11 times 10? 12 times 10?

ELEVEN.

55. When granulated sugar costs eleven cents a pound, how much must you pay for two pounds? for three pounds?

56. In Mr. Hall's orchard there are five rows of trees, and eleven trees in each row; how many trees are there altogether?

57. Find the cost of six pounds of dried apples at eleven cents per pound.

58. Mr. Rogers earns eleven dollars a week; how much does he earn in five weeks? in eight weeks?

59. How many are once eleven? twice eleven? 3 times 11? 4 times 11? 5 times 11? 6 times 11? 7 times 11? 8 times 11? 9 times 11? 10 times 11? 11 times 11? 12 times 11?

TWELVE.

60. There being twelve eggs in a dozen, how many eggs are there in two dozen? in three dozen?

61. Mary picked four quarts of blueberries; and sold them for twelve cents a quart; how much did she earn?

62. If there are twelve pence in one shilling, how many pence are there in six shillings?

63. Farmer Thorp goes to market and back, a distance in all of twelve miles, five times a week; how many miles does he travel in a week?

64. If there are twelve brimstone matches in one card, how many are there in six cards? in eight cards?

65. Mrs. Martin works for twelve cents an hour; how much does she earn by ten hours' work?

66. How many are once twelve? twice twelve? 3 times 12? 4 times 12? 5 times 12? 6 times 12? 7 times 12? 8 times 12? 9 times 12? 10 times 12? 11 times 12? 12 times 12?

B. Questions: The Multiplication Sign (×).

1. 2 times 1 are how many? 2 times 2? 2 times 3? 2 times 4? 2 times 5? 2 times 6? 2 times 7? 2 times 8? 2 times 9? 2 times 10?

2. 3 times 1 = ? means 3 times 1 equals what? or 3 times 1 are how many?

	3 times 1 = ?	3 times 6 = ?
	3 times 2 = ?	3 times 7 = ?
	3 times 3 = ?	3 times 8 = ?
	3 times 4 = ?	3 times 9 = ?
	3 times 5 = ?	3 times 10 = ?
3.	4 times 1 = ?	4 times 6 = ?
	4 times 2 = ?	4 times 7 = ?
	4 times 3 = ?	4 times 8 = ?
	4 times 4 = ?	4 times 9 = ?
	4 times 5 = ?	4 times 10 = ?
4.	5 times 1 = ?	5 times 6 = ?
	5 times 2 = ?	5 times 7 = ?
	5 times 3 = ?	5 times 8 = ?
	5 times 4 = ?	5 times 9 = ?
	5 times 5 = ?	5 times 10 = ?

5. To save labor, the word "times" is represented by the sign ×, called the "sign of multiplication." Thus, 6 × 1 means "6 times 1," or "6 multiplied by 1."

	6 × 1 = ?	6 × 5 = ?	6 × 8 = ?
	6 × 2 = ?	6 × 6 = ?	6 × 9 = ?
	6 × 3 = ?	6 × 7 = ?	6 × 10 = ?
	6 × 4 = ?		
6.	7 × 1 = ?	7 × 5 = ?	7 × 8 = ?
	7 × 2 = ?	7 × 6 = ?	7 × 9 = ?
	7 × 3 = ?	7 × 7 = ?	7 × 10 = ?
	7 × 4 = ?		
7.	8 × 1 = ?	8 × 5 = ?	8 × 8 = ?
	8 × 2 = ?	8 × 6 = ?	8 × 9 = ?
	8 × 3 = ?	8 × 7 = ?	8 × 10 = ?
	8 × 4 = ?		
8.	9 × 1 = ?	9 × 5 = ?	9 × 8 = ?
	9 × 2 = ?	9 × 6 = ?	9 × 9 = ?
	9 × 3 = ?	9 × 7 = ?	9 × 10 = ?
	9 × 4 = ?		
9	10 × 1 = ?	10 × 5 = ?	10 × 8 = ?
	10 × 2 = ?	10 × 6 = ?	10 × 9 = ?
	10 × 3 = ?	10 × 7 = ?	10 × 10 = ?
	10 × 4 = ?		

C. Miscellaneous Problems.

1. If one orange is worth as much as three apples, how many apples are four oranges worth?

2. What cost three yards of cloth at five dollars a yard?

3. If a man travels three miles in an hour, how many miles will he travel in six hours?

4. What cost seven hundred-weight of sugar at twelve dollars a hundred-weight?

5. If it takes four bushels of wheat to make a barrel of flour, how many bushels will it take to make seven barrels?

6. If horse-car tickets cost six cents apiece, how much must I pay for twelve of them?

7. My newspaper costs me three cents a day; what is that by the week of six days?

8. If cranberries are twelve cents a quart, what must I pay for eight quarts?

9. What do seven barrels of cider-vinegar cost, at eleven dollars a barrel?

10. How much do four tubs of butter come to, at eight dollars a tub?

11. What do six reams of paper cost, at five dollars per ream?

12. Find the cost of eight pairs of shoes at three dollars a pair.

13. If a man travels five miles in an hour, how many miles will he travel in nine hours?

14. There is an orchard consisting of ten rows of trees, and nine trees in each row; how many trees are there in the orchard?

15. On a chess-board there are eight rows of squares, and eight squares in each row; how many squares are there on the board?

16. In one quart there are two pints; how many pints are there in three quarts? in six quarts?

17. In one gallon there are four quarts; how many quarts are there in three gallons? in five gallons? in seven gallons?

18. If a quart of kerosene oil costs five cents, what will a gallon cost?

19. A person bought two oranges at four cents apiece, and seven lemons at two cents apiece; how much did the whole cost?

20. William's mother gave him twenty cents, and told him to buy two quarts of beans; if the beans cost eight cents a quart, how much change did William receive?

21. If a stage runs seven miles an hour, how far will it run in nine hours?

22. If a man can earn twelve dollars a week, how much can he earn in six weeks?

23. If a farmer pays ten dollars a year for the use of one acre of land, how much should he pay for five acres?

24. If the interest of one dollar is six cents a year, what would be the interest of ten dollars for the same time?

25. If the interest of one dollar is six cents for one year, what would be the interest of it for two years? for three years? for seven years?

26. Mrs. Jones has bought four yards of cotton cloth at nine cents a yard, and has given to the salesman a fifty-cent piece; how many cents should she receive in change?

27. If three men can do a piece of work in two days, how many days would it take one man to do it?

28. If a quantity of food will serve three men for five days, how many men would it serve for one day?

29. Farmer Higgins sold five bushels of beans at two dollars a bushel, and six bushels of turnips at a dollar a bushel; how much did he receive in all?

30. If six men can do a piece of work in seven days, how many men would it take to do it in one day?

31. If a quantity of food will keep five men seven days, how many days would it serve one man?

32. A man had a piece of work to perform which seven men could do in nine days, but it was necessary that the whole should be completed in one day; how many men did he have to employ?

33. Mr. Ford has eight boxes of soap: in each box there are twelve bars, and he expects to make a cent a bar by selling them; how much does he expect to make in all?

34. How much do two quires of paper, at twelve cents a quire, and a package of envelopes, at fifteen cents, come to?

35. Two men start from the same place and travel different ways; one travels two miles in an hour, the other three miles in an hour; how far apart will they be in one hour? in two hours? in four hours?

36. Two men start from the same place and travel the same way; one travels at the rate of two miles an hour, the other at the rate of four; how far apart will they be in one hour? in two hours? in four hours?

37. A milkman bought a ten-quart can of milk for fifty cents, and sold it at seven cents a quart; how much did he make?

38. In the school-room there are six rows of seats, and seven seats in each row: four of the seats are empty; how many pupils are there in the room?

39. If I have four five-pound boxes of butter and sell the butter at a profit of six cents a pound, how much do I make on the whole?

40. Two men start out at the same instant from two towns and walk towards each other: one walks at the rate of four miles an hour, and the other at the rate of three miles an hour. The men meet in two hours; how far apart are the towns?

41. A lady bought three yards of cashmere for two dollars a yard, seven yards of silk for three dollars a yard, five yards of ribbon for four dollars in all, and some crape for two dollars; how much did the whole cost? She paid four ten-dollar bills; how much should she receive back again?

42. There are 12 inches in a foot, and 3 feet in a yard; how many inches are there in a yard?

43. There are 4 gills in a pint, 2 pints in a quart, and 4 quarts in a gallon; how many gills are there in a gallon?

D. The Multiplication Table.

The multiplication table for 2 written out in full.

Once 2 are 2.	7 times 2 are 14.
2 times 2 are 4.	8 times 2 are 16.
3 times 2 are 6.	9 times 2 are 18.
4 times 2 are 8.	10 times 2 are 20.
5 times 2 are 10.	11 times 2 are 22.
6 times 2 are 12.	12 times 2 are 24.

MULTIPLICATION TABLE.

	2 are	3 are	4 are	5 are	6 are	7 are	8 are	9 are	10 are	11 are	12 are
Once	2	3	4	5	6	7	8	9	10	11	12
2 times	4	6	8	10	12	14	16	18	20	22	24
3 times	6	9	12	15	18	21	24	27	30	33	36
4 times	8	12	16	20	24	28	32	36	40	44	48
5 times	10	15	20	25	30	35	40	45	50	55	60
6 times	12	18	24	30	36	42	48	54	60	66	72
7 times	14	21	28	35	42	49	56	63	70	77	84
8 times	16	24	32	40	48	56	64	72	80	88	96
9 times	18	27	36	45	54	63	72	81	90	99	108
10 times	20	30	40	50	60	70	80	90	100	110	120
11 times	22	33	44	55	66	77	88	99	110	121	132
12 times	24	36	48	60	72	84	96	108	120	132	144

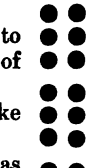
SECTION V.

Division.

A. Practical Problems bringing in the divisors 2, 3, 4, *etc., in order.*

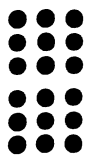
TWO.

1. How many pears at two cents apiece can you buy for four cents? for six cents?
2. If you should divide up eight counters into lots, each containing two counters, how many of these lots would there be?
3. A farmer has eight oxen; how many yoke of oxen has he?
4. There are ten stockings which Sarah has knit this winter; how many pairs of stockings has she knit?
5. How many two-cent postage-stamps can I buy with a dime and a two-cent piece?
6. How many rows of 2 dots each must you count in order to get 6? 8? 14?
7. How many times 2 are 6? 8? 14? 24? 18? 20?
8. Six tens are how many times two tens? How many times 20, then, is 60? 80? 140?



THREE.

9. How many peaches at three cents apiece can you buy for six cents? for nine cents?
10. How many rows of 3 dots each must you count in order to get 9? 12? 15? 18?
11. If a man travels three miles an hour, how many hours will it take him to go twelve miles? eighteen miles?
12. How many yards of cloth at three dollars a yard can you buy for twenty-one dollars? for thirty dollars?
13. How many oranges at three cents apiece *can I buy for* thirty-six cents?



14. Emma received thirty-three cents for the three quarts of blueberries which she had picked; how much was this a quart?

15. How many times 3 are 6? 9? 12? 15? 18? 21? 24? 27? 30? 33? 36?

16. How many times three tens are twelve tens? How many times 30, then, is 120? How many times 30 is 180?

FOUR.

17. Make on your slate (or with counters) a number of rows of dots with four dots in each row; how many of these rows must you count in order to get twelve dots? twenty dots? How many times 4, then, is 12? 20?

18. If 4 dollars will buy one pair of boots, how many pairs will eight dollars buy? sixteen dollars?

19. How many pounds of chalk at 4 cents a pound can be bought for 24 cents?

20. If you had 20 cents, how many cakes could you buy at 4 cents apiece?

21. If you have a squad of 36 men and wish to divide them up into files of four, how many files can you make?

22. How many times is 4 contained in 8? 12? 16? 20? 24? 28? 32? 36? 40? 44? 48?

FIVE.

[The following problems may be illustrated by rows of dots or counters, as above.]

23. How many packages of torpedoes at five cents a package can you buy for ten cents? for fifteen cents?

24. How many rows of five counters each must you count in order to get fifteen? twenty? twenty-five?

25. If you had twenty dollars, how many yards of cloth at five dollars a yard could you buy for the money?

26. At five for a cent, how many cents will thirty marbles cost? forty marbles?

27. How many five-cent nickels is a fifty-cent piece worth?

28. How many times is 5 contained in 10? 15? 20? 25? 30? 35? 40? 45? 50? 55? 60?

SIX.

29. How many rows of dots, with six in each row, must you count in order to get eighteen dots? thirty dots? How many times 6, then, is 18? 30?

30. If a horse can go six miles an hour, how many hours will it take him to go twelve miles? 24 miles?

31. If flounders are six cents a pound, how many pounds can you buy for eighteen cents? for thirty-six cents?

32. If you read six pages in your book each day, how many week-days will it take to read forty-eight pages?

33. Mrs. Brown bought some flower-pots for six cents apiece; how many could she get for fifty-four cents?

34. How many times 6 is 12? 24? 36? 42? 48? 54? 60? 66? 72?

SEVEN.

35. How many rows of seven each must you count in order to get fourteen? twenty-one? thirty-five?

36. If milk is worth seven cents a quart, how many quarts ought there to be in a fifty-six cent can?

37. If there are seven apple-trees in each row of Mr. Kane's orchard and forty-nine trees in all, how many rows are there?

38. If Mr. Wood puts up his oatmeal in seven pound bags, how many bags can he fill with sixty-three pounds of meal?

39. How many times 7 is 14? 21? 28? 35? 42? 49? 56? 63? 70? 77? 84?

EIGHT.

40. Find out by counting how many rows of eight each must be taken to make sixteen? twenty-four?

41. If fire-crackers cost 8 cents a bunch, how many

bunches can you buy for twenty-four cents? for thirty-two cents?

42. If a vessel can sail eight miles an hour, how many hours will it take her to sail sixteen miles? forty miles?

43. How many squads of eight each can you make out of sixty-four soldiers?

44. If there are eight quarts in one peck, how many pecks are there in a bushel (32 quarts)?

45. How many times is 8 contained in 16? 24? 32? 40? 48? 56? 64? 72? 80? 88? 96?

NINE.

46. How many groups of nines must you count in order to get eighteen? twenty-seven? forty-five?

47. In a certain orchard there are 36 trees in rows, with nine trees in each row; how many rows of trees are there?

48. A man paid 27 dollars for some sheep; if each sheep cost 9 dollars, how many did he buy?

49. If salted cod-fish costs nine cents a pound, how much can you buy for seventy-two cents?

50. If a man works nine hours a day, how many days must he work in order to get through a job which will take fifty-four hours?

51. How many times 9 is 18? 27? 36? 45? 54? 63? 72? 81? 90? 99? 108?

TEN.

52. How many ten-cent pieces is a fifty-cent piece worth?

53. If a bar of soap costs ten cents, how many bars can you get for forty cents? for eighty cents?

54. If the rent of one acre of land per year is ten dollars, how many acres can be hired for thirty dollars? for seventy dollars?

55. If a box of parlor-matches costs ten cents, how many boxes can you buy for sixty cents? for one dollar (100 cents)?

56. How many times 10 is 20? 30? 50? 70? 40? 60? 120? 100? 90? 110?

ELEVEN.

57. If blueberries cost eleven cents a quart, how many quarts can you buy for thirty-three cents? for fifty-five cents?

58. Mr. Childs has 77 pounds of butter which he wishes to send to market in a number of boxes which he has bought; if each box holds 11 pounds, how many boxes will be needed in all?

59. Mr. Trow has 99 currant-bushes in his garden, and 11 in each row; how many rows of bushes has he?

60. If you are measuring with a pole eleven feet long, how many lengths must you take for 88 feet? for 132 feet?

61. How many times 11 is 44? 33? 88? 121? 55? 132? 110? 77? 99? 22? 66?

TWELVE.

62. Farmer Jackson gathered forty-eight eggs from his hen-house last week; how many dozen were there?

63. If there are twelve inches in a foot, how many feet long is a table which measures thirty-six inches?

64. If a quart of cranberries costs 12 cents, how many quarts can you get for 72 cents? for 96 cents?

65. A hotel-keeper wants 60 cakes of toilet-soap to put into the rooms in his hotel; if there are twelve cakes in a box, how many boxes must he buy?

66. How many times is 12 contained in 24? 48? 36? 72? 96? 120? 108? 60? 84? 132? 144?

B. Questions.

1. 4 are how many times 1? 2? 4?
2. 6 are how many times 1? 2? 3? 6?
3. How many 2's are there in 8? how many 4's?
4. 9 are how many times 3?
5. 10 are how many times 2? 5?
6. 12 are how many times 2? 3? 4? 6?

7. 14 are how many times 2? 7?
8. 15 are how many times 3? 5?
9. 16 are how many times 2? 4? 8?
10. 18 are how many times 2? 3? 6? 9?
11. 20 are how many times 2? 4? 5? 10?
12. 21 are how many times 3? 7?
13. 22 are how many times 2? 11?
14. 24 are how many times 2? 3? 4? 6? 8? 12?
15. How many 5's are there in 25? in 35? in 45? in 55? in 65? in 60? in 50? in 40? in 30?
16. 27 are how many times 3? 9?
17. 28 are how many times 7? 4?
18. 30 are how many times 3? 6? 10?
19. How many pounds of rice, at 8 cents a pound, can you buy for 32 cents?
20. How many 4's in 32? how many 8's? how many are 2 times 16?
21. How many bananas, at 7 cents apiece, can you buy for 35 cents? how many 7's in 35?
22. 36 are how many times 3? 4? 6? 9? 12?
23. 40 are how many times 4? 5? 8? 10?
24. How many 11's are there in 44? in 66? in 33? in 88? in 55? in 77? in 99?
25. 42 are how many times 6? 7?
26. At 4 cents apiece, how many oranges can be bought for 44 cents?
27. 45 are how many times 9?
28. 48 are how many times 4? 6? 8? 12?
29. If a glass of soda-water costs 7 cents, how many glasses can I buy for 49 cents?
30. How many 10's are there in 50? 60? 90? 80? 70?
31. 54 are how many times 6? 9?
32. 56 are how many times 7? 8?

[We are said to *divide* one number by another when we find how many times the first contains the second. Thus, 24 is 6 times 4, or 24 divided by 4 is 6.]

33. Divide 28 by 4; 35 by 7; 36 by 9; 42 by 6.

34. Divide 60 by 5; by 6; by 10; by 12.

[To save labor, the words "divided by" are often represented by the sign $\div$, called the "sign of division." Thus, $32 \div 8$ stands for "32 divided by 8."]

35. How many are $35 \div 5$? $42 \div 7$? $48 \div 12$?

36. Write the proper number after the sign of equality in each of the following cases: $63 \div 7 =$; $64 \div 8 =$; $66 \div 11 =$; $72 \div 8 =$; $72 \div 6 =$.

37. Divide 63 70 72 72 77 80 81
by 9 7 9 12 7 8 9.

38. 84 is how many times 7? 12?

39. How much is $88 \div 8$? $88 \div 11$? $90 \div 9$? $96 \div 8$? $96 \div 12$? $99 \div 9$? $99 \div 11$?

40. Divide 100 108 108 110 121 132
by 10 9 12 10 11 11.

41. How much is $120 \div 12$? $132 \div 12$? $144 \div 12$?

C. Miscellaneous Problems.

1. A boy has thirty cents which he wishes to spend for fire-crackers and marbles; if he buys as many bunches of fire-crackers, at eight cents a bunch, as the money will allow, and lays out the remainder in marbles, at five for a cent, how many marbles does he get? how many bunches of fire-crackers?

2. How many spools of thread, at four cents apiece, can you buy for twenty-five cents? how many cents will you have left over?

3. Mary has thirty-two cents, with which she wishes to buy Florida oranges for a friend who is ill; if the oranges cost five cents apiece, how many can she buy? how many cents will she have left over?

4. John and Samuel fished from the dock and caught twenty-six sculpins, which they carried to the lobsterman for bait. The lobsterman agreed to give them a lobster for every eight sculpins, and a cent for every pair of sculpins left over; how many lobsters did they get? how many cents?

5. Farmer Benson sends his milk to the cheese-factory in twelve-quart cans. One day his cows gave sixty-four quarts of milk; how many cans could he fill? how many quarts were left over?

6. How many times is 8 contained in 20, and how many are left over?

7. Mr. Beal puts up his honey for market in eight-pound boxes: he has seventy-five pounds of honey in all, and fills as many boxes as he can, and keeps what is left over; how many boxes of honey does he send to market? how many pounds does he keep?

8. Mrs. White carried her eggs to the grocer's, and sold them for twenty cents a dozen; with the proceeds she bought as many pounds of sugar as she could for eleven cents a pound, and brought the rest of her money home: if she had three dozen eggs, how many pounds of sugar did she get, and how much money had she left over?

9. How many times is 11 contained in 36, and how many are left over?

10. A man is to walk from Boston to Portland, a distance of 106 miles: if he can walk 20 miles a day, how many days must he spend on the journey, and how many miles must he walk on the last day?

11. Mr. Hutchins sells his fresh cucumbers for twenty cents a dozen; one morning he picked fifty-four cucumbers from his vines, sold as many whole dozens as possible, and kept the rest: how much money did he receive, and how many cucumbers did he keep?

12. Henry, Richard, and Margaret picked 45 quarts of wild blackberries, and filled with them as many eight-quart boxes as they could to send to the city: they sold the berries that they had left over for nine cents a quart; how much of the money ought each to receive?

13. There are 31 days in January; how many full weeks are there, and how many days are left over?

14. Three feet make a yard; how many yards are there in 28 feet?

SECTION VI.

Written Addition and Subtraction.

This section and the following relate to Written Arithmetic, and may be studied now or introduced later, at the option of the teacher.

A. Questions on Notation, with Explanations.

1. What number is represented by 36?

Answer: Thirty-six.

What does the 3 stand for? the 6?

Answer: 36 = 3 tens + 6 units; the 3, then, stands for the whole number of tens in thirty-six, and the 6 for the units left over.

2. What do the figures in 42 represent? in 69? in 58? in 17? in 23?

3. What does the right-hand figure of a number denote? what does the figure to the left of the right-hand figure denote?

Since the right-hand figure denotes *units*, and the figure to the left of it denotes *tens*, we say that the first figure from the right occupies the *units' place*, and that the second figure from the right occupies the *tens' place*.

4. What is the number that has 8 in the units' place and 6 in the tens' place? (*Answer:* Sixty-eight.) 9 in the units' place and 7 in the tens' place? 5 in the tens' place and 2 in the units' place? 4 in the tens' place and 0 in the units' place?

Tens.	Units.
6	8
7	9
5	2
4	0

5. What number is represented by 857?

Answer: Eight hundred and fifty-seven.

What do the different figures stand for?

Answer: 857 = 8 hundred + 5 tens + 7 units; the 8, then, stands for the whole number of hundreds, the 5 for the whole number of tens left over, and the 7 for the extra units remaining after taking away both the hundreds and the tens.

6. What do the different figures represent in 614? in 891? in 412? in 508? in 320?

7. What does the figure to the left of the tens' place denote?

8. Which place, counting from the right, is the hundreds' place?

9. What is the number that has 6 in the unit's place, 8 in the tens' place, and 4 in the hundreds' place? (*Answer:* Four hundred and eighty-six.) 9 in the tens' place, 7 in the hundreds' place, and 2 in the units' place? 6 in the units' place, 0 in the tens' place, and 8 in the hundreds' place? 6 in the hundreds' place, 0 in the tens' place, and 0 in the units' place?

Hundreds.	Tens.	Units.
4	8	6
7	9	2
8	0	6
6	0	0

10. The number 320 stands for 32 tens, but it is often convenient to call 10 tens a "hundred," so that 32 tens is 3 hundred and 2 tens, or three hundred and twenty. Similarly we call, for convenience, 10 hundred (1000) a "thousand"; 20 hundred (2000) 2 "thousand"; 30 hundred (3000), 3 "thousand"; 3600, then, stands for 36 hundred, or 3 *thousand and* 6 *hundred;* 13200 for 132 hundred, or 13 *thousand and* 2 *hundred;* 168362 for 168 *thousand* 3 *hundred and* 62.

Thousands.		Hundreds.	Tens.	Units.
3	2	8	9	6	8
	2	8	6	0	8
1	7	0	4	3	2
	1	6	0	7	3
1	0	0	7	6	8
9	8	2	7	4	2

Write in words each of the numbers in the adjacent column. First number: Three hundred and twenty-eight *thousand* nine *hundred* and *sixty-eight.*

11. What figures stand for thousands in each of the above numbers? what figure stands for hundreds? for tens? for units?

12. Counting from the right, what places do the figures that represent thousands occupy?

In order to make it easier to read a number, it is usual to separate by a comma the figures that denote thousands from the figure that denotes hundreds; thus, in 198,642 we place a comma between the 8 and the 6, and read 198 *thousand,* 6 hundred and 42.

13. Read: 986,432; 16,768; 270,432; 100,708; 908,550; 7,487; 2,008; 68,052; 111,684; 777,777.

14. Express by figures the numbers:

Seventy-seven thousand, six hundred and eighty-four.

Thirty-four thousand, and fifty-six.

Four hundred and seventeen thousand, six hundred.

Seven hundred and fourteen thousand, six hundred.

Six hundred thousand, and two.

Three hundred and ninety-one thousand, six hundred and twelve.

Fifty-seven thousand, one hundred and nine.

Nine hundred thousand, four hundred and three.

15. 1,000,000, or 1 thousand thousand, is usually called a "million." 16,000,000, or 16 thousand thousand, may be called, then, 16 millions; 186,000,000, or 186 thousand thousand, may be called 186 millions.

	Millions.	Thousands.	Hundreds. Tens. Units.
186 million,	186,	000,	000.
432 thousand,		432,	000.
186 million, 432 thousand, and 792,	186,	432,	792.
799 million, and 684,	799,	000,	684.
896 million, and 479 thousand,	896,	479,	000.

Counting from the right, what places do the figures that represent *millions* occupy?

16. What do we call the first place from the right? the second place? the third place? the fourth, fifth, and sixth places? the seventh, eighth, and ninth places?

17. Read the numbers:

982,461,007	111,111,111	2,760,286
698,231,770	232,008,674	16,888,888
842,000,689	11,400,800	769,000,001

18. Express by figures the numbers:

Six million, seventy-five thousand, and four.

Three hundred and six million, and forty thousand.

Five million, six hundred and seventeen thousand, and forty-three.

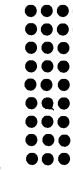
Four hundred and sixty million, and twenty-seven.

Seventy-three million, forty-one thousand, eight hundred and nineteen.

19. Express by figures the numbers:
Nine hundred million, one hundred and seven.
Eighty-six million, seven hundred.
Thirteen hundred and eighty-six million, four hundred and ninety-seven thousand, three hundred and sixty-two.

B. Written Addition, with Practical Illustrations.

1. How many complete rows of ten each can you make out of 32 counters? how many counters will be left over?

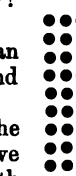


Answer: 3 complete rows of ten, and there will be two extra counters.

2. How many complete rows of ten each can you make with 24 counters? how many counters will be left over?

3. How many complete rows of ten counters each can you make with 49 counters? 83 counters? 61 counters? 17 counters? How many extra counters will there be in each of these cases?

4. How many counters must you have in order to make 5 complete rows and have 3 counters over? 7 rows and 2 counters over?



5. How many complete rows of ten each can you make with 32 counters *and* 24 counters, and how many counters will be left over?

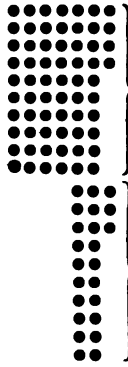
Answer. It is easy to see, by looking at the diagrams given above, that with 32 counters we should have 3 rows and 2 extra counters; with 24 counters we should have 2 rows and 4 extra counters; with 32 *and* 24 counters we should then have 3+2 rows and 2+4 extra counters, or 5 rows and 6 counters.

6. How many complete rows of ten counters each can you make with 42 counters +53 counters? how many counters will be left over?

7. Mr. Smith paid 23 cents for a piece of cheese, and 36 cents for some butter; how much did he pay in all?

8. Mrs. Jones paid 64 cents for veal, and 23 cents for vegetables; how much did she pay for her dinner?

Let us solve this problem by the aid of counters.

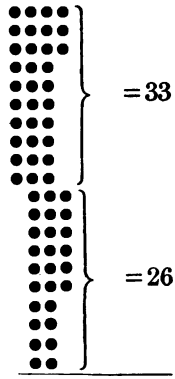


64 counters = 6 rows of tens + 4 counters.

23 counters = 2 rows of tens + 3 counters.

In all there are 8 rows of tens and 7 extra counters, or 87 counters; therefore, 64 + 23 = 87, and Mrs. Jones paid 87 cents for her dinner.

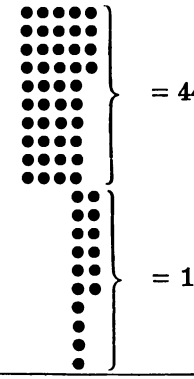
9. How many are 33 + 26? 44 + 16?



= 33

= 26

5 tens + 9 = 59.



= 44

= 16

5 tens + 10 = 6 tens = 60.

10. Add	35	41	97	23	84	16	80	41
to	43	18	22	56	11	23	12	28
Answers:	78	59	119	79	95	39	92	69

11. George earned 27 cents by carrying papers, and 35 cents by building fires; how much did he earn in all?

We may obtain the answer by the aid of counters:

In the two incomplete rows there are 7+5 or 12 counters, which make one row, with 2 counters left over: all together, then, there are (1+5) 6 rows of tens+2 counters, or 62 counters.

Therefore, 27+35=62, and George earned 62 cents.

12. How many are 36+28?

The extra counters make 1 row and 4 counters: all together, then, there are 6 rows and 4 counters.

Therefore, 36+28=64.

13. Add	23	24	16	35	83	44	76	31
to	18	67	17	48	29	39	16	49
Answers:	41	91	33	83	112	83	92	80

WITHOUT COUNTERS.

14. How many are 74 + 68?

74 = 7 tens + 4 units.
68 = 6 tens + 8 units.

74 + 68 = 13 tens + 12 units
= 14 tens + 2 units = 142.

Or, more briefly,

74
68
142

8 and 4 are 12, or 1 ten + 2 over; we set down the 2 and save the 1 ten. 1 ten (the one that was saved) and 6 are 7 and 7 are 14 tens; we set down the 14 to the left of the two, and have 142 for an answer.

15. Add 56 to 28

Answer: 84

8 and 6 are 14; we set down the 4 and save the 1. 1 (the 1 that was saved) and 2 are 3, and 5 are 8; we set down the 8 to the left of the 4, and get 84 for our answer.

16. Add	25	89	57	84	76
to	65	98	75	29	56
Answers:	90	187	132	113	132

17. How many are 269 + 328?

269 = 2 hundred + 6 tens + 9 units.
328 = 3 hundred + 2 tens + 8 units.

269 + 328 = 5 hundred + 8 tens + { 17 units / or 1 ten + 7 units }
= 5 hundred + 9 tens + 7 units
= 597.

18. How many are 684 + 767?

684 = 6 hundred + 8 tens + 4 units.
767 = 7 hundred + 6 tens + 7 units.

684 + 767 = 13 hundred + 14 tens + 11 units
= 13 hundred + 15 tens + 1 unit
= 14 hundred + 5 tens + 1 unit
= 1451.

19. Add	793	184	Write out your work.
to	848	678	

20. Add 768
to 857
1625

7 and 8 are 15; we set down the 5 and save the 1. 1 and 5 are 6, and 6 are 12; we set down the 2 and save the 1. 1 and 8 are 9, and 7 are 16; we set down the 16, and have for an answer 1625.

21. Add
to

	689	972	439	139
	763	684	698	984
Answers:	1452	1656	1137	1123
277	858	900	237	642
166	686	768	508	899

22. Add 667
276
108
188
1239

8 and 8 are 16, and 6 are 22, and 7 are 29; we set down the 9 and save the 2. 2 (the 2 that was saved) and 8 are 10, and 7 are 17, and 6 are 23; we set down the 3 and save the 2. 2 and 1 are 3, and 1 are 4, and 2 are 6, and 6 are 12; we set down the 12. We have for an answer 1239.

23. Add

284	326	522	683	
355	719	799	793	
123	268	655	573	
618	400	321	498	
1380	1713	2297	2547	
687	322	623	60	864
921	888	798	730	777
760	666	231	815	298
208	419	23	402	476

24. America was discovered by Columbus in 1492, and the War for Independence began 283 years afterwards; in what year did this war begin?

25. In July, 1776, the Declaration of Independence was made; 87 years afterwards the Battle of Gettysburg was fought: find the date of the Battle of Gettysburg.

26. A butcher bought 3 fat oxen: the first weighed 1214 pounds, the second 1406 pounds, and the third 1384 pounds; how much live-meat had he in all?

27. A planter sold 3 bales containing 1236 pounds of cotton to one man, and 8 bales containing 3356 pounds to another; how many pounds did he sell in all?

28. In 1840 there were, according to the census, 17,069,453 inhabitants in the United States; in 1850 there were 6,122,423 more; what was the number of inhabitants of the United States in 1850?

29. Mr. Peabody received last year from Australia 1,152,614 pounds of wool by one ship, 863,223 pounds by another ship, and 1,689,826 pounds by a third; how many pounds of wool did he receive in all?

30. Mr. Ashburton paid 1656 dollars for his new stable, 250 dollars for a pair of horses, 84 dollars for harnesses, and 220 dollars for a carriage; how much did he pay in all?

31. George's new suit cost 28 dollars, his hat 3 dollars, his shoes 5 dollars, and the rest of his outfit 59 dollars; how much did the whole cost?

32. In the first volume of Meyer's Lexicon there are 1098 pages, and in the second there are 1068 pages; how many pages are there in both volumes?

33. In October, 1883, there were in Harvard College 972 students, in the Law School 146, in the Medical School 243, and in the other departments of the University 165; how many students were there in all?

34. The following is a summary of the students in Yale College in October, 1883:

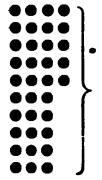
In Theology, 99; in Medicine, 31; in Law, 69; in the Scientific School, 212; in the School of Fine Arts, 49; in the Undergraduate Department, 612; in the Graduate Department, 30. How many students were there in all?

35. In the year 1875, 37,436,368 tons of hard coal were mined in the German Empire and 10,367,686 tons of soft coal; how many tons of coal were raised in all that year?

36. In the year 1879 there were mined in Great Britain 133,808,000 tons of coal; in the United States, 59,808,398 tons; in Germany, 42,631,729 tons; in France, 17,104,485 tons; in Belgium, 14,839,175 tons; in Austria-Hungary, 14,378,605: how many tons were mined in all?

C. Written Subtraction, with Practical Illustrations.

1. Take away 8 counters from 35 counters and how many will remain?



= 35 counters = 3 rows + 5 counters.

To take away 8 counters we first take away the 5 extra counters, and then going to the next row we keep on taking counters away until in all we have taken away 8; counting those that remain, we find that there are 2 rows + 7 counters or 27 counters.

35 less 8 are how many?

2. Take away 18 counters from 35 counters and how many will remain?

The 35 counters are shown in the last question.

18 counters = 1 row + 8 counters.

After taking away first the 8 counters, and then the 1 row, we find by counting that 17 counters remain.

35 less 18 are how many?

3. From	82	71	95	76
take	23	45	37	59
Answers:	59	26	58	17

Illustrate by counters.

WITHOUT COUNTERS.

4. How many are 48 less 32?

48 = 4 tens + 8 units.
32 = 3 tens + 2 units.

48 − 32 = 1 ten + 6 units = 16. *Answer.*

5. From	68	99	84	74
take	42	76	31	32
Answers:	26	23	53	42

6. How many are 53 less 18?

53 = 5 tens + 3 units.
18 = 1 ten + 8 units.

We cannot take 8 units from 3 units; we therefore take one of the 5 tens and add it to the 3 units.

53 = 4 tens + 13 units.
18 = 1 ten + 8 units.

53 − 18 = 3 tens + 5 units = 35. *Answer.*

7. How many are 97 less 68?

97 = 8 tens + 17 units.
68 = 6 tens + 8 units.

97 − 68 = 2 tens + 9 units = 29. *Answer.*

8. From	77	97	73	36
take	38	49	58	28
Answers:	39	48	15	8

9. A farmer raised 89 bushels of potatoes: he kept 18 bushels for his family, and sold the rest; how many bushels did he sell?

10. A man who owed a bill of 96 dollars, paid 20 dollars at one time, and 19 dollars at another; how much did he then owe?

11. Mr. Jackson raised 78 dollars' worth of hay above what he needed to feed to his stock: he sold this hay to the grocer and received in part pay 29 dollars' worth of flour; how much money should he receive?

12. Mr. Weston started to walk from Portsmouth to

Boston, a distance of 56 miles: he walked 34 miles on the first day; how many miles had he left to go on the second day?

13. Mr. Fowler, the mason, brought 73 bricks in his wheelbarrow to finish the arch he was building: he used only 57 of them; how many had he left?

14. Mr. Knapp's parlor is 97 inches high: the wainscoting above which the room is to be papered is 33 inches high. Into what lengths must the paper be cut?

15. Mr. Lanman sent 67 books to be bound: the binder made mistakes in the lettering of 19 of them; how many were lettered correctly?

16. 683 less 421 are how many?

From 683 = 6 hundred + 8 tens + 3 units,
take 421 = 4 hundred + 2 tens + 1 unit.

683 − 421 = 2 hundred + 6 tens + 2 units
= 262. *Answer.*

17. From	896	764	972	541
take	784	451	630	321
Answers:	112	313	342	220

18. How many are 684 less 296?

From 684 = 6 hundred + 8 tens + 4 units,
take 296 = 2 hundred + 9 tens + 6 units.

We cannot take 6 units from 4 units; we therefore take one of the 8 tens and add it to the 4 units, so that the problem will read:

From 684 = 6 hundred + 7 tens + 14 units,
take 296 = 2 hundred + 9 tens + 6 units.

We cannot take 9 tens from 7 tens; we therefore take one of the 6 hundred and add it to the 7 tens; and our problem is now as follows:

From 684 = 5 hundred + 17 tens + 14 units,
take 296 = 2 hundred + 9 tens + 6 units.

684 − 296 = 3 hundred + 8 tens + 8 units
= 388. *Answer.*

19. How many are 762 less 381?

From 762 = 7 hundred + 6 tens + 2 units,
take 381 = 3 hundred + 8 tens + 1 unit.

or

From 762 = 6 hundred + 16 tens + 2 units,
take 381 = 3 hundred + 8 tens + 1 unit.

762 − 381 = 3 hundred + 8 tens + 1 unit
= 381. *Answer.*

20. From	811	623	762	436
take	422	237	478	184
Answers:	389	386	284	252

21. From 974 take 783.

We may state briefly what we do thus: 3 from 4 gives 1; we set down the 1 in the units' column. One of the 9 hundreds added to the 7 tens gives 17 tens, and 8 tens from 17 tens are 9 tens; we set down the 9 in the tens' column. 7 hundred from the remaining 8 hundred is 1 hundred; we set down the 1 in the hundreds' column.

974
783
191

We have, then, 191 for an answer.

22. From	864	726	419
take	471	233	126
Answers:	393	493	293

23. From 921 take 246.

921
246
675

One of the 2 tens added to the 1 unit gives 11 units, and 6 from 11 are 5; we set down the 5 in the units' column. One of the 9 hundred added to the remaining 1 ten gives 11 tens, and 4 tens from 11 tens are 7 tens; we set down the 7 in the tens' column. 2 hundred from the remaining 8 hundred are 6 hundred; we set down the 6 in the hundreds' column.

We have, then, 675 for an answer.

24. From	684	555	911	722
take	296	166	223	333
Answers:	388	389	688	389

25. From 842 take 468.

We may state our work very briefly thus:

8 from 12 are 4;
6 from 13 are 7;
4 from 7 are 3.

842
468
374

26. From 633
take 216
Answer: 417

6 from 13 are 7
1 from 2 is 1
2 from 6 are 4.

27. From 846
take 674
Answer: 172

4 from 6 are 2
7 from 14 are 7
6 from 7 is 1.

28. From 811
take 322
Answer: 489

2 from 11 are 9
2 from 10 are 8
3 from 7 are 4.

29. From 600
take 268
Answer: 332

8 from 10 are 2
6 from 9 are 3
2 from 5 are 3.

30. America was discovered by Columbus in the year 1492; how many years ago was that?

31. The Pilgrims landed at Plymouth in 1620; how many years ago was that? how many years after the discovery of America by Columbus?

32. The Chicago, Burlington and Quincy Railroad built a wooden bridge, using in the construction 113,150 feet of timber. 63,217 feet were used in the supports; how many were used in the superstructure?

33. In 1876 London had 3,489,428 inhabitants, and New York only 1,064,272; how many more inhabitants had London than New York?

34. From	7692	78423	12345
take	4734	57684	7890

35. Harvard College was founded in the year 1636; how many years ago was that?

36. The length of the earth's equator is 40,070,368 meters, but the length of a meridian is only 40,003,423 meters; how much longer is the equator than a meridian?

37. The English war-ship Devastation displaces 9,190 tons of water, the German man-of-war Preussen 6,748 tons; how much more does the Devastation displace than the Preussen?

38. The population of the United States in 1830 was 12,866,020; in 1870, 38,925,598; what was the increase of population during these forty years?

D. United States Money.

10 mills = 1 cent.
10 cents = 1 dime.
10 dimes or 100 cents = 1 dollar.

1. The sign $ is often used to stand for dollars; thus, 6 dollars is usually written $6.

Read $12; $16; $24; $196.

2. Write 48 dollars; 96 dollars; 132 dollars; 2137 dollars; using the dollar mark, $, in each case.

3. A man bought a sail-boat for $325, and paid $88 for a new set of sails, and $32 for having the boat painted; how much did the whole cost?

4. Mr. Kresus had $1000 in the Lydian Five-Cents Savings Bank, but he drew out $432 to pay for a lot of land; how much was there left in the bank?

5. A dealer bought some flour for $642, and sold it at a gain of $97; what did he get for his flour?

6. A man bought a lot of land for $672, and paid $167 for having it graded: he then sold the land at a profit of $125; how much did he get for the land?

7. Write 37 dollars and 24 cents.

In such a case as this it is customary to write the number denoting cents after the number denoting dollars and to separate the two numbers by a period, thus: $37.24.

In the same way we write

48 dollars and 79 cents,	$48.79
22 dollars and 53 cents,	$22.53
22 dollars and 43 cents,	$22.43
22 dollars and 33 cents,	$22.33
22 dollars and 23 cents,	$22.23
22 dollars and 13 cents,	$22.13
22 dollars and 03 cents,	$22.03.

Notice that when the number of cents is less than 10, we fill out the empty tens' place by a zero. Thus we do not write 22 dollars and 3 cents $22.3, but $22.03.

8. Read $49.13 ; $117.88 ; $109.76 ; $104.70 ; $99.04 ; $98.10 ; $92.07 ; $100.01 ; $417.62 ; $3189.25 ; $4281.50.

Notice carefully that $625 stands for 625 dollars, but that $6.25 stands for 6 dollars and 25 cents.

9. Write in figures :

Four hundred and seven dollars.
Fifty-six dollars and thirty-seven cents.
Two hundred and forty-three dollars and five cents.
Five thousand six hundred and forty dollars and nine cents.
Eight thousand seven hundred and eighty-three dollars.
Eighty-seven dollars and eighty-three cents.
Six hundred dollars and eight cents.

10. 85 cents may be written $0.85.

Read $0.33 ; $0.07 ; $0.69 ; $0.38 ; $0.02.

11. Write in the same way 48 cents ; 83 cents ; 78 cents ; 50 cents ; 4 cents.

12. How many cents are there in $8.32 ?

Answer : 832 cents.

13. How many cents are there in $6.19 ? in $41.31 ? in $64.09 ? in $3142.27 ?

14. Show that

183 cents are equivalent to $ 1.83.
192 cents are equivalent to $ 1.92.
7189 cents are equivalent to $71.89.
3787 cents are equivalent to $37.87.
3601 cents are equivalent to $36.01.

15. Add \$37.13 to \$43.81. *Answer:* \$80.94.

In doing such examples as this, write the numbers in a column, taking care to write cents under cents and dollars under dollars; the numbers may then be added as if the periods were not there.

16. Add

	\$38.29	\$47.82	\$117.98
	\$26.18	\$32.51	\$ 21.08
	\$64.47	\$80.33	\$139.06
			\$780.98
\$316.47	\$4132.07	\$397.09	\$ 0.18
\$283.32	\$ 8.17	\$ 71.93	\$ 31.50

17. Mrs. Wentworth paid \$128 for a chamber-set, \$46 for mattresses and bed linen, \$48.50 for a carpet, \$24 for curtains, and \$32.38 for other articles needed in her spare chamber; what was the cost of furnishing the room?

18. A New England farmer wished to move out West, so he sold his farm and his stock and with the proceeds went to Kansas. He sold his two horses for \$246, his oxen for \$98, his sheep for \$136, his pigs for \$24.50, and his poultry for \$16.48; how much did he get in all from the sale of his stock?

19. On the morning of the 8th of May Mr. Brown had only \$186.14 in the bank, but in the course of the day he deposited \$112 in bills, and checks amounting to \$347.32; how much had he then in the bank?

20. Add

\$4268.07	\$138.06	\$6842.16
\$6842.16	\$291.98	\$6843.27
\$7999.99	\$236.72	\$7234.82
\$2316.11	\$972.36	\$6124.86
\$7000.98	\$842.16	\$5555.55
\$1119.72	\$777.77	\$3232.39
\$2942.15	\$666.66	\$9998.99
\$3715.35	\$545.53	\$6849.76
\$1872.89	\$379.23	\$2332.29
\$6792.91	\$192.98	\$6974.84

SECTION VII.

Written Multiplication and Division.

This section, together with the preceding, may be brought in now or introduced later, at the option of the teacher.

A. Multiplication. Examples and Problems, with Remarks and Explanations.

1. What will four pounds of coffee cost at thirty-two cents a pound?

Four pounds of coffee will cost four times as much as one pound, or four times thirty-two cents.

$$32 = 3 \text{ tens} + 2 \text{ units.}$$
$$4 \text{ times } 32 = 4 \text{ times } 3 \text{ tens} + 4 \text{ times } 2 \text{ units}$$
$$= 12 \text{ tens} + 8 \text{ units}$$
$$= 120 + 8 = 128.$$

Four pounds of coffee, then, will cost 128 cents or $1.28.

2. Mr. Wood, the grocer, makes a profit of 21 cents on every bag of meal that he sells; what will be his profit on 4 bags?

3. How many are 5 times 41?

4. If beefsteak is 23 cents per pound, what will 3 pounds cost?

5. How many are 4 times 52?

$$52 = 5 \text{ tens} + 2 \text{ units.}$$
$$4 \text{ times } 52 = 4 \text{ times } 5 \text{ tens} + 4 \text{ times } 2 \text{ units}$$
$$= 20 \text{ tens} + 8 \text{ units}$$
$$= 200 + 8 = 208.$$

To save space we may write our work thus:

52
4
$8 = 4 \times 2$ units.
$200 = 4 \times 5$ tens.
$208 = 4 \times 52$.

208 for an answer.

Or, still more briefly:

We may briefly describe our work as follows: 4 times 2 are 8; we write down the 8: 4 times 5 are 20; we write down the 20 to the left of the 8 and get

52
4
208

6. Multiply	42	61	53	82
by	3	4	3	5
	126			

7. When we have to perform an example in multiplication it is sometimes convenient to call the number which we are asked to multiply the MULTIPLICAND, the number by which we are to multiply the MULTIPLIER, and the result obtained by our work the PRODUCT.

Thus, in example 5 the *multiplicand* is 52, the *multiplier* is 4, and the *product* is 208.

Name the multiplicand, the multiplier, and the product in the examples of 6.

8. If one barrel of sugar costs 19 dollars, how much will 5 barrels cost?

The answer will be 5 times 19 dollars, and we are to find out how many dollars this will make.

19 = 1 ten + 9 units.

5 times 19 = 5 times 1 ten + 5 times 9 units
= 5 tens + 45 units
= 5 tens + 4 tens + 5 units
= 9 tens + 5 units
= 95.

Five barrels of sugar, then, at 19 dollars a barrel will cost 95 dollars.

9. If a railway train goes 36 miles in an hour, how far will it go in 3 hours?

10. How many are 5 times 64?

11. Mr. Reardon gets 25 cents an hour for his work; how much does he get for 8 hours' work? how much for 4 hours' work?

12. How many are 6 times 78?

78 = 7 tens + 8 units.

6 times 78 = 6 times 7 tens + 6 times 8 units
= 42 tens + 48 units
= 42 tens + 4 tens + 8 units
= 46 tens + 8 units
= 460 + 8
= 468. *Answer.*

To save space we may write our work thus:

$$\begin{array}{r} 78 \\ \times\ 6 \\ \hline 48 = 6 \times 8 \text{ units.} \\ 420 = 6 \times 7 \text{ tens.} \\ \hline 468 = 6 \times 78. \end{array}$$

13. Multiply 29 by 7

$63 = 7 \times 9$ units.
$140 = 7 \times 2$ tens.
Answer: $203 = 7 \times 29$.

Multiply 98 by 6

$48 = 6 \times 8$.
$540 = 6 \times 9$ tens.
Answer: $588 = 6 \times 98$.

How many are 36×5? 43×8? 57×9?

14. Multiply 84 by 6

24
480
Answer: 504

Instead of writing down the 24 and the 480 separately, we may add them together in our heads as we go on and say: 6 times 4 are 24, or 2 tens and 4 units; we set down the 4 and save the 2 tens to add in with other tens. 6 times 8 tens are 48 tens; these with the 2 tens saved over make 50 tens, and we write down the 50 to the left of the 4.

$$\begin{array}{r} 84 \\ \times\ 6 \\ \hline 504 \end{array}$$

15. Multiply 96 by 4

384

4 times 6 are 24; set down the 4 and save the 2. 4 times 9 are 36; add in the 2 that were saved and set down the result, 38, to the left of the 4.

16. Multiply 85 by 7

595

7 times 5 are 35; set down the 5 and save the 3. 7 times 8 are 56; add in the 3 and set down the result, 59, to the left of the 5.

17. Multiply

49	28	37	65
by 7	8	5	9
343	224	185	585

18. What will six pounds of chocolate cost at 38 cents per pound?

19. At 19 cents a pound, what will a roasting piece of beef weighing 8 pounds cost?

20. If 7 men can dig a ditch in 24 days, how long would it take 1 man to do it?

21. If one barrel of vinegar holds 32 gallons, how much vinegar can be contained in 6 barrels? how much in 8 barrels?

22. If one barrel of flour contains 196 pounds, how many pounds will there be in four barrels?

$196 = 1 \text{ hundred} + 9 \text{ tens} + 6 \text{ units}$

$4 \times 196 = 4 \times 1 \text{ hundred} + 4 \times 9 \text{ tens} + 4 \times 6 \text{ units}$

$= 4 \text{ hundred} + 36 \text{ tens} + 24 \text{ units}$

$= 400 + 360 + 24$

$= 784.$

Four barrels of flour, then, will contain 784 pounds.

We may save space by arranging our work thus:

196
4

$24 = 4 \times 6$ units.
$360 = 4 \times 9$ tens.
$400 = 4 \times 1$ hundred.

$784 = 4 \times 196.$

23. How many are 3 times 218?

218
3

$24 = 3 \times 8$ units.
$30 = 3 \times 1$ ten.
$600 = 3 \times 2$ hundred.

Answer: $654 = 3 \times 218.$

Multiply	341	208	121	610
by	6	3	8	6

24. Mr. Blake paid three dollars and twenty-five cents apiece for six chairs; how much did the whole cost him?

25. If one barrel of flour is worth seven dollars and forty cents, how much are six barrels worth?

26. Multiply 243 by 8

	24
	320
	1600
	1944

Instead of writing down the 24, the 320, and the 1600, separately, we may add them together as we go on with our work, and say: 8 times 3 are 24; we set down the 4 and save the 2 tens. 8 times 4 tens are 32 tens, and these with the 2 tens saved over make 34 tens, or 3 hundred and 4 tens; we set down the 4 tens and save the 3 hundred. 8 times 2 hundred are 16 hundred, and these with the 3 hundred make 19 hundred, which we set down to the left of the figures that we already have.

	243
	8
	1944

In like manner

Multiply	416	123	321	643
by	7	4	5	2

27. Multiply 134 by 7.

After you have learned how to do such examples as those of No. 26 *correctly* and *quickly*, you may tell as briefly as possible what you actually do in multiplying two numbers together, thus:

134	
7	7 times 4 are 28; we set down the 8 and carry the 2. 7 times 3 are 21, and 2 are 23; we set down the 3 and carry the 2. 7 times 1 are 7, and 2 are 9; we set down the 9.
938	

28. Multiply

by	2196	1419	6021	8206
	5	3	6	8
Answer:	10980			

29. 10 times 36 are how many?

We know that 10 times 36 are the same as 36 times 10. Therefore 10 times 36 = 36 tens or 360.

10 times 49 are how many? *Answer:* 490.
10 times 64 are how many? *Answer:* 640.
10 times 90 are how many? *Answer:* 900.
10 times 16 are how many? *Answer:* 160.
10 times 83 are how many? *Answer:* 830.

You have multiplied different numbers by 10; write down each of the above numbers, and underneath it the answer thus:

49	64	90	16	83
490	640	900	160	830

What do you do, then, when you multiply a number by 10?

30. 20 times 37 are how many?

10 times 37 are 370.

Twice this or 20 times 37 are 2 times 370 or 740.

Or 20 times 37 = 37 times 20
= 37 times 2 tens
= 74 tens
= 740.

31. Multiply	68	49	36	51
by	30	40	50	70
Answers:	2040	1960	1800	3570.

Notice that the first answer may be obtained by placing a zero after 3 times 68, the second by placing a zero after 4 times 49, the third by placing a zero after 5 times 36, and so on.

32. Multiply	46	46	46	46
by	100	200	300	700
Answers:	4600	9200	13800	32200
Multiply	64	82	76	98
by	600	500	800	400
Answers:	38400	41000	60800	39200.

Notice that the first answer may be obtained by writing two zeros after 1×46, the second by writing two zeros after 2×46, etc., etc.

33. How many are 24 times 36?

$$\begin{array}{r} 36 \\ 24 = 20 + 4. \\ \hline 144 = 36 \times 4. \\ 720 = 36 \times 20. \\ \hline 864 = 36 \times 24. \end{array}$$

34. How many are 18 times 23?

$$\begin{array}{r} 23 \\ 18 = 10 + \ 8. \\ \hline 184 = 23 \times \ 8. \\ 230 = 23 \times 10. \\ \hline 414 = 23 \times 18. \end{array}$$

35. If butter is worth 38 cents a pound, what is the value of a tub containing 35 pounds?

Answer: 35 times 28 cents, or 980 cents.

36. What is the cost of 24 hoes at 58 cents each?

37. Mr. Jones bought 32 hens at 96 cents apiece; how much did he pay for them all?

38. If a man, who is draining a field, can dig 48 feet of ditch in one day, how much can he dig in 22 days?

39. I bought 42 boxes of soap each containing 18 bars; how many bars of soap were there in all?

40.

Multiply	98	69	73	46
by	47	39	13	54

41. After you have learned to multiply correctly, you may shorten your written work as much as possible.

Multiply 49 by 27.

$$\begin{array}{r} 49 \\ 27 \\ \hline 343 = 49 \times \ 7. \\ 980 = 49 \times 20. \\ \hline 1323 = 49 \times 27. \end{array}$$

or, more briefly,

$$\begin{array}{r} 49 \\ 27 \\ \hline 343 \\ 98 \\ \hline 1323. \end{array}$$

You may omit the zero after the 98, if you are careful to show that the 98 represents 98 *tens*, by moving the number one step to the left, so that the 8 may stand in the column of tens and not in the column of units.

42. Multiply 76 by 52 = 152 + 380 = 3952.

2 times 6 are 12; we set down the 2 and carry the 1. 2 times 7 are 14, and the 1 make 15. 5 times 6 are 30; we set down the zero in the column of tens and carry the 3. 5 times 7 are 35, and the 3 make 38, etc., etc.

43. Multiply 71 48 64 49 83
by 19 12 72 38 25

44. If a barrel of flour contains 196 pounds, how many pounds are there in 48 barrels?

196
48

$1568 = 196 \times 8.$
$7840 = 196 \times 40.$
$9408 = 196 \times 48.$

or, more briefly,

196
48
1568
784
9408.

45. If coal is worth four dollars and sixty-four cents a ton by the cargo, what will a cargo of three hundred and twenty-seven tons cost?

If one ton costs 464 cents, 327 tons will cost 327 times 464 cents. We are, then, to multiply 464 by 327.

$327 = 300 + 20 + 7.$

464
327

$3248 = 464 \times 7.$
$9280 = 464 \times 20.$
$139200 = 464 \times 300.$
$151728 = 464 \times 327.$

or, more briefly,

464
327
3248
928
1392
151728.

The whole cargo will cost, then, 151728 cents, or 1517 dollars and 28 cents.

46. Multiply 318 | Multiply 406
by 425 | by 719
1590 | 3654
636 | 406
1272 | 2842
Answer: 135150. | *Answer:* 291914.

Multiply 873 223 417 803
by 614 196 391 516

47. Multiply 456 by 204.

$204 = 2 \text{ hundred} + 0 \text{ tens} + 4$
$= 200 + 4.$

456			456
204		or, more	204
1824 = 456 × 4.		briefly,	1824
91200 = 456 × 200.			912
93024 = 456 × 204.			93024.

Multiply	471	822	583	649
by	306	507	109	408
Answer:	144126			

48. In multiplying one number by another, you have learned to write down the number to be multiplied (the "multiplicand") over the multiplier; and, beginning at the right, to multiply the multiplicand by each figure of the multiplier separately; and to write down your results one over another, so that the *right-hand figure* of each shall be *directly under that figure of the multiplier* which was used to obtain this result. By adding together the results thus arranged you get the product required. Thus:

Multiply	8164
by	4821
	8164
	16328
	65312
	32656
	39358644

In this case 8164 is the multiplicand and 4821 is the multiplier. The result obtained by multiplying 8164 by the 1 is 8164, and is set down so that its right-hand figure 4 is directly under the 1. In the same way the result, 16328, obtained by multiplying 8164 by the 2, is set down so that its right-hand figure is directly under the 2, etc., etc.

Multiply	3142	7162	6824	5691
by	693	8237	9036	2007

49. The railroad from Boston to New York by way of Springfield is 233 miles long: there are 1760 yards in a mile; how many yards of track are there between Boston and New York?

50. The track from Darlington to Weston is laid with

steel rails weighing 130 pounds to the yard of track: the distance between these cities is 98 miles; how many pounds of steel are there in all?

51. In a certain town there are 1842 acres planted in wheat: if each acre yields, on an average, 28 bushels, how many bushels will be raised this year in the town?

52. If a bale of cotton weighs on the average 408 pounds, how many pounds are there in 714 bales?

53. In a certain mill are 623 operatives, and each gets on the average 36 dollars a month; how much is paid to the operatives in a month? in a year?

54. In a certain regiment there are 986 men, who receive 28 pounds of meat apiece per month; how many pounds of meat are needed to supply the whole regiment for a month?

55. Find the cost of 680 acres of land in the West at 13 dollars and 25 cents per acre.

56. A railway pays two dollars and twenty-five cents a car-load for gravel; how much must it pay for 2875 loads?

B. Division: Examples and Problems, with Remarks and Explanations.

1. If one barrel of apples costs 3 dollars, how many barrels can you buy for 96 dollars?

We are to divide 96 by 3.

$$96 = 9 \text{ tens} + 6.$$
$$96 \div 3 = 9 \text{ tens} \div 3 + 6 \div 3$$
$$= 3 \text{ tens} + 2$$
$$= 32.$$

You can, then, buy 32 barrels of apples, at 3 dollars a barrel, for 96 dollars.

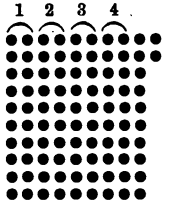
Divide 84 by 4; 48 by 2; and 63 by 3.

2. A man who wished to pay a bill of 2 dollars and 48 cents in a neighboring town, bought the money's worth of 2-cent stamps, and sent the stamps in a letter to his creditor; how many stamps did he buy? We are to divide 248 by 2.

$$248 = 2 \text{ hundreds} + 4 \text{ tens} + 8.$$
$$248 \div 2 = 2 \text{ hundreds} \div 2 + 4 \text{ tens} \div 2 + 8 \div 2$$
$$= 1 \text{ hundred} + 2 \text{ tens} + 4$$
$$= 124. \quad \textit{Answer.}$$

Divide 369 by 3; 484 by 4; and 624 by 2.

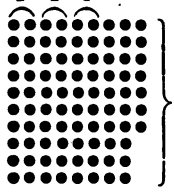
3. If you divide 92 counters among 4 boys, how many will each receive?



There are evidently rows enough to give each boy 2 full rows; now dividing the remaining 1 row and 2 counters, or 12 counters, among the 4 boys, each will receive 3 counters.

Therefore, each boy will receive in all 2 rows + 3 counters, or 23 counters.

4. If you divide 87 counters among 3 girls, how many will each receive?



There are evidently rows enough to give each girl 2 full rows; now dividing the remaining 2 rows and 7 counters, or 27 counters, among the 3 girls, each will receive 9 counters.

Therefore, each girl will receive in all 2 rows + 9 counters, or 29 counters.

5. Gertie, Herbert, and Edith went out chestnuting

together, agreeing to divide equally all that they should find: when they reached home they counted out their chestnuts into heaps of ten each, and found that they had 8 heaps and 4 chestnuts over, that is, 84 in all; how many chestnuts should each receive?

If you solve this problem by the aid of counters, using a counter to represent a chestnut, you will find that each should receive 28 chestnuts.

6. Mr. Smith, Mr. Jones, and Mr. Robinson cultivated together a vegetable garden: beside supplying their families with vegetables, they sold 57 dollars' worth and divided the money; how much did each receive?

7. When we have to perform an example in division, it is sometimes convenient to call the number which we are asked to divide the DIVIDEND, the number by which we are to divide the DIVISOR, and the result obtained by our work the QUOTIENT.

Thus, in example 3 the *dividend* is 92, the *divisor* is 4, and the *quotient* is 23.

Name the dividend, the divisor, and the quotient in the examples of 1 and 2.

8. Divide 96 by 4.

96 = 9 tens + 6 units.

4 goes into 9 tens 2 tens times and leaves an extra ten, which added to the 6 units makes 16 units. 4 goes into 16 units 4 times; therefore 4 goes into 96

2 tens times + 4 times or 24 times.

We can save space by arranging our work thus:

4)96(20 + 4.
80 = 4 × 20.
16 remainder.
16 = 4 × 4.
0 remainder.

We may say 4 goes into 9 tens 2 tens times; we set down the 2 tens to the right of the dividend and subtract 2 tens times 4 or 80 from the dividend. The remainder is 16. 4 goes into 16, 4 times; we set down the 4 to the right of the dividend and subtract 4 times 4 from our 16. The remainder is nothing. Our quotient, then, is 20 + 4 or 24.

In this example which is the dividend? which the divisor? which the quotient?

9. Divide 78 by 3.

3)78(20+6.
60 = 3 × 20.
18 = remainder.
18 = 3 × 6.
0 = remainder.
Answer: 26.

Divide 52 by 4.

4)52(10+3.
40
12
12
0
Answer: 13.

Divide 36 by 2; 54 by 3; 72 by 4; and 95 by 5.

10. Divide 532 by 4.

4)532(100+30+3.
400 = 4 × 100.
132 = remainder.
120 = 4 × 30.
12 = remainder.
12 = 4 × 3.
0

4 goes into 5 hundred 1 hundred times; we set down 100 in the quotient and subtract 1 hundred times 4 or 400 from 532 with 132 for a remainder. 4 into 13 tens goes 3 tens times; we set down 30 in the quotient and subtract 30 times 4 or 120 from 132 with 12 for a remainder. 4 into 12 goes 3 times; we set down the 3 in the quotient and subtract 3 times 4 or 12 from 12 with nothing for a remainder. Our quotient, then, is

100+30+3, or 133.

Divide 423 by 3; 738 by 6; 314 by 2; and 854 by 7.

11. Divide 556 by 4.

4)156(30+9.
120 = 4 × 30.
36 = remainder.
36 = 4 × 9.
0

4 is not contained in 1; we therefore put the 1, which stands for 1 hundred, with 5 tens making 15 tens. 4 into 15 tens goes 3 tens or 30 times; we write the 30 in the quotient and subtract 30 times 4 or 120 from 156 with 36 for a remainder. 4 into 36 goes 9 times; we write the 9 in the quotient and subtract 9 times 4 from 36 with nothing for a remainder. Our quotient, then, is 30+9, or 39.

Divide 432 by 6; 801 by 9; 336 by 4; and 525 by 7.

12. If a bunch of fire-crackers costs 8 cents, how many bunches can you buy for $1.28?

13. If a pound of brown sugar costs 9 cents, how many pounds can you buy for $2.34?

14. A man bought 8 acres of land for $912; what was that an acre?

15. How many miles a day must an Atlantic steamer make, if she is to cross the ocean, a distance of 2870 miles, in 7 days? how many miles a day if she is to cross in 8 days.

16. If 4 quarts make a gallon, how many gallons are there in 376 quarts?

17. Sarah wants very much a fine writing desk for which the dealer asks $3.87. If she gets 9 cents a quart for picking berries, how many quarts must she pick in order to earn enough to buy the desk?

18. Divide 792 by 6.

6)792 | 100	We may shorten our	6)792 | 1
600 | 30	work by omitting some	6 | 3
192 | 2	of the figures, thus:	19 | 2
180		18
12		12
12		12
0		

Answer: 132.

We may also write all the figures of our quotient on one line, as in the following:

Divide 833 by 7.	Divide 968 by 4.
7)833(119.	4)968(242.
7	8
13	16
7	16
63	8
63	8

Divide 865 by 9; 784 by 8; 644 by 7; and 795 by 5.

19. If a barrel of flour costs 8 dollars, how many barrels can be bought for 184 dollars?

20. The 4.30 express from Boston to New York makes the distance of 234 miles in 6 hours; how many miles an hour must the train cover?

21. How many yards of cotton cloth at 9 cents a yard can be bought for $486?

22. George gets 9 cents an hour for weeding onions; how long will it take him to earn $1.35?

23. Where the divisor is a small number, it is usual to do in one's head a large part of the work of dividing.

Divide 725 by 5.

5)725

145 = quotient.

5 into 7 goes once with a remainder of 2; we set down the 1 under the 7 and put the remainder with the next figure of the dividend, making 22. 5 into 22 goes 4 times, with a remainder of 2; we set down the 4 as the second figure of the quotient, and put the remainder with the next figure of the dividend, making 25. 5 into 25 goes 5 times, with no remainder. 5, then, is the last figure of the quotient, and $725 \div 5 = 145$.

Show that $595 \div 5 = 119$; $552 \div 4 = 138$.

24. Divide 632 by 4.

4)632

158

4 into 6 goes once with 2 over. 4 into 23 goes 5 times with 3 over. 4 into 32 goes 8 times with no remainder.

Divide 185 by 5.

5)185

37

5 into 1 will not go; 5 into 18 goes 3 times with 3 over. 5 into 35 goes 7 times.

Show that the work is correct in each of the following examples:

6)192	7)651	4)1264	5)7285	9)374504508
32	93	316	1457	41611612

25. The distance from London to Edinburgh by the Great Northern Railway and its connections is 405 miles; how many miles an hour must the daily express, which covers the distance in 9 hours, make?

26. Where the divisor is large, it is necessary to write out in full most of the work.

Divide 496 by 16.	Divide 322 by 14.
16)496(31.	14)322(23.
48	28
16	42
16	42

27. Sometimes we can tell only by trial what the different figures of the quotient are.

Divide 2001 by 29.

29)2001(69.
174
261
261

Here, in order to get the first figure of the quotient, we have to find, by trial, how many times 29 is contained in 200.

If we try 7 we find that $7 \times 29 = 203$; therefore 7 is too large and we try 6. In the same way we have to find *by trial* how many times 29 is contained in 261.

Show that $1862 \div 38 = 49$, and that $4446 \div 78 = 57$.

28. I bought a chest of tea containing 18 pounds for \$12.96; how much was that a pound?

29. A man bought a piece of land for \$672, and agreed to pay for it at the rate of \$15 per month; how long will it take him to pay the whole?

30. A cattle dealer bought a drove of 27 oxen for \$972; how much did each ox cost him?

31. Mr. Bartlett bought 14 boxes of soap containing 18 bars each for \$30.24; how much was that a bar?

32. If a pound of coffee costs 38 cents, how many pounds can be bought for \$20.90?

33. A grain dealer received a car-load of wheat weighing 7920 pounds. A bushel of wheat weighs 60 pounds; how many bushels were there in the car-load?

34. Mr. Dodge's yearly salary is \$1728; how much is that a month?

35. If there are 12 inches in a foot, how many feet are equivalent to 7776 inches?

36. Show that $524778 \div 1761 = 298$, and that $77312 \div 4832 = 16$.

C. Miscellaneous Problems.

1. Mr. Smith bought a piano for $336, and agreed to pay $12 a month until the whole should be paid; how many payments had he to make?

2. Divide 9936 by 27; 22464 by 468; 2304 by 128.

3. There are 640 acres in a square mile; how many square miles are there in a township containing 8320 acres?

4. Mr. Hood, the plumber, bought of a dealer 895 feet of gas pipe at 8 cents a foot, 4 valves at $1.50 each, 24 plugs at 11 cents apiece, and $3 worth of solder; what was the whole amount of Mr. Hood's bill?

5. Farmer Jackson sold at market one day 12 pounds of butter at 32 cents a pound, 14 bushels of potatoes at 65 cents a bushel, 9 dozen eggs at 26 cents a dozen, and 2 bushels of onions at $1 a bushel; how much did he receive in all?

6. I bought a lot of wild land containing 114 acres at $7 per acre, paid $86 in taxes, and sold it for $9 per acre; how much did I make by the transaction?

7. I bought 173 barrels of sugar for $3633, and sold it for $22.75 per barrel; how much did I make? How much did I make on each barrel?

8. I bought 18 quires of paper at 12 cents a quire, and 10 packages of envelopes at 9 cents a package. I gave the stationer a ten-dollar bill; how much ought I to have received in change?

9. The circumference of each fore wheel of a carriage is 8 feet, and of each hind wheel 12 feet; how many more turns will the fore wheels make than the hind wheels in going 5280 feet or 1 mile? How many more turns in going from Boston to Worcester, a distance of 44 miles?

10. The year 1884, being a leap-year, contained 366 days; how many hours did it contain? If I slept 8 hours each day, how many hours did I sleep in the whole year? If I had read 2 hours each day, at the rate of

23 pages an hour, how many pages should I have read in the whole year? how many books of 276 pages each?

11. Find the cost of each of the following items and add the results together:

25 bushels of potatoes at $0.60 per bushel.
128 pounds of sugar at $0.11 per pound.
138 pounds of coffee at $0.32 per pound.
84 Winchester rifles at $28 apiece.
14 pairs of oars at $2.85 per pair.
148 gallons of kerosene oil at $0.16 per gallon.
648 pounds of tea at $0.82 per pound.

12. Find the cost of the articles which Mr. Slade bought for his store when he was last in Boston.

He bought:

25 yards of silk at $1.37 per yard.
25 yards of silk at $2.10 per yard.
48 yards of ribbon at $0.28 per yard.
214 yards of cotton cloth at $0.08 per yard.
96 yards of cashmere at $0.82 per yard.

And a number of small articles which came to $118.25.

13. The number of square miles in Texas is 265,780; in Massachusetts is 8,315. Into how many states of the size of Massachusetts could Texas be divided, and how many square miles would be left over?

Multiply			Divide		
14.	69834 by	9761.	28.	19798336 by	8432.
15.	76497 by	2864.	29.	11604186 by	6798.
16.	9426 by	984.	30.	3222720 by	1080.
17.	16214 by	4962.	31.	55494492 by	18006.
18.	7698 by	1111.	32.	167376 by	132.
19.	8432 by	5555.	33.	178316384 by	10832.
20.	9012 by	7006.	34.	681649674 by	69834.
21.	2984 by	1080.	35.	219087408 by	76497.
22.	6798 by	9807.	36.	9275184 by	984.
23.	8432 by	2348.	37.	80453868 by	16214.
24.	9821 by	2484.	38.	8552478 by	1111.
25.	16462 by	10832.	39.	46839760 by	5555.
26.	1268 by	132.	40.	63201126 by	7006.
27.	18006 by	3082.	41.	24395364 by	9821.

SECTION VIII.

Fractions.

A. One Half, One Third, etc., in Order.

HALVES.

1. When anything is divided into two equal parts one of the parts is called *one half* of the thing.

What is each of the two equal parts of this pear called?

Answer: One half of a pear.

How many halves of a pear are there in a whole pear?

Answer: Two halves of a pear.

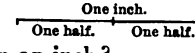
2. What is each of the two equal parts of this peach called?

Answer: One half of a peach.

How many halves of a peach are there in a whole peach?

Answer: Two halves of a peach.

3. Draw on your slate a line one inch long, and divide it into two equal parts; what is one of the parts called?



How many half inches are there in an inch?

4. Edward Stevens and John Robinson went out sailing together; the wind was high and they needed something to tie on their hats with; Edward found in his pocket two yards of twine, half of which he gave to John. How much did Edward keep, and how much did he give to John?

[Illustrate by a string two yards long.]

5. What is one half of 2?

6. Bertram had 4 cents, half of which he gave to his sister Alice; how many cents did he keep? how many cents did he give to Alice?

7. What is one half of 4? what part of 4 is 2?

8. It takes 12 inches to make a foot. How many inches are there in half a foot?

[Illustrate with a foot-rule.]

9. What is one half of 12?

10. It takes 16 ounces to make a pound. How many ounces are there in half a pound?

11. What is one half of 16?

12. If a man can ride a bicycle 14 miles in an hour, how far can he ride in half an hour?

13. What is one half of 14?

14. How many cents are there in one half-dollar? how many half-dollars are there in a dollar?

15. What is one half of 100?

16. If a yard of cotton cloth costs 16 cents, what will half a yard cost?

17. What is one half of 16?

18. Joseph bought half a pound of raisins for 11 cents; what can I buy a pound for?

19. 11 is one half of what number?

Answer: If 11 is one half, then 2 halves, or a whole, will be 2 times 11, or 22. Therefore 11 is one half of 22.

20. If half a yard of elastic costs 4 cents, what will a whole yard cost?

21. 4 is one half of what number?

22. If half a pound of maple sugar costs 8 cents, what will 2 pounds cost?

Answer: If half a pound costs 8 cents, 2 halves, or a pound, will cost 2 times 8 cents, or 16 cents. If one pound costs 16 cents, 2 pounds will cost 2 times 16 cents, or 32 cents.

23. A carpenter charges $2 for half a day's work; what must I pay him for 3 days' work?

24. 1 is one half of what number?

Of what number is 2 the half? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? [Explain in each case, as in Question 19.]

25. It takes 2 pints to make a quart. What part of a quart is a pint?

26. It takes 4 quarts to make a gallon. What part of a gallon will a two-quart measure hold?

27. What part of 6 is 3? ●●● ●●●

Answer: 3 is contained in 6 two times; therefore 3 is one half of 6.

28. If you can buy a barrel of cranberries for $10, what part of a barrel can you buy for $5? 5 is what part of 10?

29. What part of 2 is 1? of 4 is 2? of 8 is 4? of 10 is 5? of 20 is 10? of 16 is 8? of 22 is 11? of 18 is 9? of 100 is 50?

THIRDS.

1. When anything is divided into three equal parts one of the parts is called *one third* of the thing.

Thus, each one of the three equal parts into which this orange has been cut is called one third of the orange. How many thirds are there in an orange? in an apple? in anything?

2. If I cut a stick of candy into three equal parts and give one part to Joseph and the rest to Nellie, how much of the candy will Joseph have? how many thirds will Nellie have?

3. If you divide three cents equally among three boys, how much will each boy get?

What is one third of three? ●●●

4. What is one third of six? ●● ●● ●●

5. If you divide a dozen oranges among three children, how many oranges will each child get?

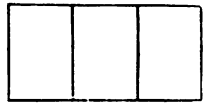
●●●● ●●●● ●●●●

What, then, is one third of twelve?

6. Mr. Smith gave his three children, George, Mary, and Tom, one of the beds in the garden, and told them that they might plant in it whatever they chose. This was the shape of the bed.

Mary wished to plant flowers, but

her brothers said that they would rather raise vegetables; so it was finally agreed that the bed should be divided into three equal parts, and that each child should take one part.



How much of the whole bed had Mary? How much had George? How much had Tom?

The whole bed was 30 feet long; how long was each part?

Tom soon got tired of taking care of his garden and gave it up to George, but the others kept on all summer. How much of the whole bed did George cultivate?

Among other things George raised four melons; he gave one of them to his father and mother and agreed to share the others equally with his brother and sister. How many melons ought Mary to have?

One morning Mary cut six roses and gave a third of them to her mother, another third to her teacher, and the rest to a sick school-mate; how many did she give to her school-mate?

7. If a yard of silk be worth three dollars, and it be cut into three equal pieces, what will one of the pieces be worth?

What, then, is one third of three dollars?

8. A man who has a ditch to dig finds after working all day that he has dug only one third of it; how many days more must he work in order to finish the ditch?

9. Farmer Higgins, who wished to raise oats in one of his fields, found that it took five bushels of seed to plant one third of the field; how many bushels must he sow in the whole field?

Five is one third of what number?

10. A foot is one third of a yard. If there are twelve inches in a foot, how many inches are there in a yard?

11. Seven is one third of what number?

Answer: If one of the three equal parts into which the number may be divided is seven, the whole number must be equal to three times seven, or twenty one. Seven, then, is one third of twenty-one.

12. Of what number is 8 one third? *Answer:* 24.

13. If one third of an acre of land rents for three dollars, how much ought I to pay for the use of a two-acre lot?

14. Eleven is one third of what number?

15. Fourteen is two thirds of what number?

Answer: If 14 is two thirds, one third would be one half of 14, or 7. The number itself, or three thirds, is, then, three times 7, or 21. Therefore 14 is two thirds of 21.

16. Eighteen is two thirds of what number? Of what number is 12 two thirds?

17. What is two thirds of 33?

18. A young man who earned $27 per month spent two thirds of his earnings, and saved the rest; how much did he save in a year?

FOURTHS.

1. If anything is divided into four equal parts, each part is called one *fourth* of the thing. Here is an orange cut into fourths. How many fourths are there in an orange? in an apple? in anything?

2. There are four quarts in a gallon; what part of a gallon is a quart?

3. There are four pecks in a bushel; what part of a bushel, then, is a peck?

4. If you divide a dollar into fourths, how many fourths will you get?

One fourth of anything is also called one *quarter* of that thing.

5. If a yard of velvet costs 4 dollars, what will one quarter of a yard cost? what will three quarters of a yard cost?

6. If you are to divide anything among four persons, what part will each receive?

7. Sarah gave one fourth of her cake to Mary, and another fourth to Hannah; how much of her cake did she give away? how much did she keep?

8. Into how many halves may an apple be divided? into how many fourths? How many fourths of an apple are there in half an apple?

9. What is one half of 4? what is two fourths of 4?

10. What is one half of 8? what is two fourths of 8?

11. What is one half of 12? what is two fourths of 12?

12. How many cents are there in half a dollar? how many cents are there in two quarters of a dollar?

13. One half is the same as how many fourths?

14. If you divide four sticks of candy among four children, how many sticks will each child receive?

What is one fourth of 4?

15. Mrs. White gave Mary a dozen peaches to share with the three school-mates who had come to see her; how many peaches did each of the four little girls have?

What is one fourth of 12? ●●● ●●● ●●● ●●●

16. What is one fourth of 8? ●● ●● ●● ●●

17. What is one fourth of 36? of 24? of 44?

18. There are sixty minutes in an hour; how many minutes are there in one quarter of an hour?

19. How many cents are there in one quarter of a dollar?

20. 5 is one fourth of what number?

Answer: The number must be such that if it be divided into four equal parts, each of those parts will be equal to 5; the whole number, then, is equal to four times 5, or 20. Therefore 5 is one fourth of 20?

21. Of what number is 8 one fourth?

22. 9 is one fourth of what number?

23. If a quarter of a yard of ribbon costs twelve cents, what will two yards cost?

24. 12 is three fourths of what number?

Answer: If 12 is three fourths of a number, one fourth must be one third of 12, or 4; four fourths, or

the number itself, will be four times 4, or 16. Therefore 12 is three fourths of 16.

25. 18 is three fourths of what number?

26. Of what number is 24 three fourths?

27. Mrs. Smith paid 75 cents for three quarters of a yard of cloth; how much was that a yard?

28. If it takes 9 days to do three fourths of a piece of work, how many days more will it take to finish the job?

29. If four bushels of corn will buy one yard of cloth, how many yards will one bushel buy? three bushels?

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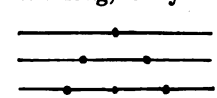
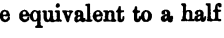
30. Draw three lines, each one inch long, on your slate. Divide the first into halves, the second into thirds, and the last into quarters. Which is longer, one half of an inch or one third of an inch? one third of an inch or one fourth of an inch? how many fourths of an inch are equivalent to a half of an inch?



FIFTHS.

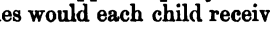
1. If anything is divided into five equal parts one of these parts is called *one fifth* of the thing. Here is a stick of candy which has been divided into five equal parts; what is one of these parts called? How many fifths of a melon are there in a whole melon? how many in two melons?



2. If five dollars will buy a box of butter, what will one fifth of a box cost? What part of a box will two dollars buy?

3. What is one fifth of five? What part of five is three? *Answer:* Three fifths of five.

4. If you were to divide ten apples equally among five children, how many apples would each child receive?



5. What is one fifth of ten?

6. What is one fifth of 15? of 25? of 30? of 40?

7. 3 is one fifth of what number?

Answer: If 3 is one fifth, five fifths, or the whole number, is 5 times 3, or 15.

8. 8 is one fifth of what number?

9. If it takes Mary 4 days to read one fifth of her new book, how many days must she spend over the whole book?

10. If 5 yards of gingham cost one dollar, how much will one yard cost?

11. Find two fifths of 30.

Solution: If 30 be divided into five equal parts one of these parts, or *one fifth* of 30, will be 6; *two fifths* of 30, then, will be two times 6, or 12.

12. Find three fifths of 40; four fifths of 25; two fifths of 45.

13. 9 is three fifths of what number?

Answer: If 9 is three fifths, one fifth must be one third of 9, or 3; five fifths, or the whole, will be five times 3, or 15. Therefore 9 is three fifths of 15.

14. 20 is four fifths of what number? Of what number is 8 two fifths?

15. If a man can dig one fifth of an acre of potatoes in a day, how many days will it take to dig the potatoes out of a field of two acres?

16. A man, who had a certain length of fence to build, found that it took him four days to put up two fifths of the fence; how many more days will it take him to finish the job?

17. Five boys hired a boat for a day's fishing, and agreed to pay the owner 60 cents; how much ought each boy to pay?

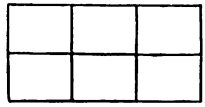
With only four boys in the boat, what would be each boy's share? with three boys? with two boys?

18. What is one half of 60? one third of 60? one fourth of 60? one fifth of 60? Is it always true that one fifth of a thing is smaller than one fourth of the same thing?

19. What is one fifth of 20? one fourth of 20?

SIXTHS.

1. When anything is divided into six equal parts one of the parts is called one sixth of the thing. This loaf of cake is divided into six equal parts or sixths.



Six feet make a fathom; what part of a fathom is one foot?

2. There are six working days in a week; what part of a week's work can a man do in one day?

What is one sixth of six? ●●●●●●

3. If you divide a dozen oranges among six children how many oranges will each child receive?

●● ●● ●● ●● ●● ●●

What is one sixth of twelve?

4. If you divide eighteen apples into six equal heaps, how many will there be in each heap? What is one sixth of 18?

5. Find one sixth of 42; of 54; of 48; of 30; of 36.

6. 7 is one sixth of what number?

7. What part of 24 is 4? of 48 is 8?

8. If one bushel of grass-seed is needed to sow one sixth of an acre of lawn, how many bushels must be used for a lawn of three acres?

9. 15 is five sixths of what number?

Answer: If 15 is five sixths, one sixth will be one fifth of 15, or 3; six sixths, or the number itself, will be six times 3, or 18. Therefore 15 is five sixths of 18.

10. 25 is five sixths of what number?

11. Mrs. Hill paid 50 cents for five sixths of a yard of lining; how much was that a yard?

12. If a box of toilet soap containing six cakes costs 72 cents, what part of 72 cents ought I to pay for one cake? for five cakes?

13. If a steamer sails 66 miles in six hours, how many miles can she sail in five hours?

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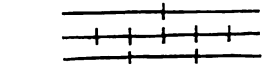
14. Draw upon your slate three lines each an inch


long and divide the first into halves, the next into sixths, and the last into thirds; measure and see how many sixths of an inch there are in half an inch, and how many sixths of an inch there are in one third of an inch.

15. What is one half of 6? one sixth of 6? three sixths of 6? How many sixths of 6 make one half of 6?

16. What is one half of 12? one sixth of 12? three sixths of 12? How many sixths of 12 make one half of 12?

17. What is one half of 18? one sixth of 18? three sixths of 18? How many sixths of 18 make one half of 18?

18. How many sixths make one half?

Solution: Anything may be divided into 6 sixths; one half of that thing, then, may be divided into one half of 6 sixths or 3 sixths; therefore 3 sixths make one half.

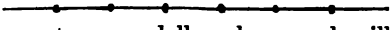
19. How many sixths of 12 make one third of 12?

20. How many sixths of 18 make one third of 18?

21. How many sixths of anything make one third of the thing?

SEVENTHS.

1. When anything is divided into seven equal parts, one of the parts is called one seventh of the thing. Here is a line divided into sevenths.



If a barrel of flour costs seven dollars, how much will one seventh of a barrel cost? two sevenths? three sevenths?

2. What is one seventh of 14? two sevenths? three sevenths? four sevenths? six sevenths?

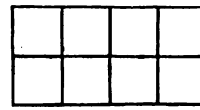
3. 8 is two sevenths of what number?

Solution: If 8 is two sevenths, one seventh would be one half of 8, or 4; seven sevenths would be seven times 4, or 28. Therefore 8 is two sevenths of 28.

4. 9 is three sevenths of what number? 20 is five sevenths of what number? of what number is 42 six sevenths?

EIGHTHS.

1. When anything is divided into eight equal parts one of the parts is called one eighth of the thing. Here is a cake of maple sugar divided into eighths.



There are eight quarts in a peck; how many quarts are there in one eighth of a peck? in three eighths of a peck? What part of a peck is five quarts?

2. What is one eighth of eight? three eighths of eight? five eighths of eight? seven eighths of eight?

●●●●●●●●

3. How many eighths of an apple are there in an apple? If you divide anything into eighths, how many parts will there be?

4. There are 16 ounces in a pound; how many ounces are there in one eighth of a pound? in three eighths of a pound? in five eighths of a pound?

5. George is only three eighths as old as his big brother, who is 24; how old is George?

6. 6 is one eighth of what number?

Answer: 6 is one eighth of 8 times 6, or 48.

7. 9 is one eighth of what number?

8. Of what number is 4 one eighth? 7? 11?

9. Farmer Brown told Patrick Flynn that if he would dig all the potatoes in a certain field he might have one eighth of them for his trouble. Patrick's share was five bushels; how many bushels were there in all?

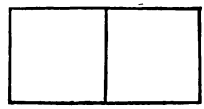
10. 15 is three eighths of what number? *Answer:* 40.

11. 21 is seven eighths of what number?

12. 45 is five eighths of what number?

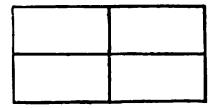
13. If seven eighths of a yard of ribbon cost 35 cents, what is the price per yard?

14. Into how many halves can you divide a cake of chocolate?



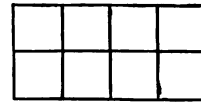
2 halves.

15. Into how many fourths can you divide the cake?



4 fourths.

16. Into how many eighths can you divide the cake?



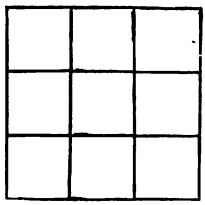
8 eighths.

17. One half of a cake contains how many fourths? how many eighths?

18. One fourth of a cake contains how many eighths?

19. What is one eighth of 16? one fourth of 16? one half of 16?

NINTHS.



1. When anything is divided into nine equal parts, one of the parts is called one ninth of the thing. Here is a square divided into ninths.

If I divide eighteen lozenges among nine little girls, how many lozenges will each get?

●● ●● ●● ●● ●● ●● ●● ●● ●●

What, then, is one ninth of eighteen lozenges?

2. What is one ninth of eighteen? two ninths of eighteen? five ninths of eighteen?

3. What is one ninth of 27? of 36? of 54?

4. Find two ninths of 45; of 81.

5. Find five ninths of 63; of 72.

6. If it takes two days to do one ninth of a piece of work, how many days will the whole job take?

7. If it takes six days to plow two ninths of a certain field, how many days more will it take to finish the work?

---

8. What is one third of 18? one ninth of 18? three ninths of 18?

9. How many ninths of 18 make one third of 18?

10. Here are two lines of equal lengths; the first is divided into thirds,
and the second into 
ninths. Measure and see how many ninths make one third.

TENTHS.

1. When anything is divided into ten equal parts, one of the parts is called one tenth of the thing. Here is a line divided into tenths.

A dime is equivalent to ten cents; how many cents are equivalent to one tenth of a dime? three tenths? seven tenths?

2. What is one tenth of 10? two tenths of 10? three tenths of 10? ten tenths of 10?

3. Into how many tenths can you divide an apple? an orange? anything?

4. Mr. Blake sold seven tenths of his crop of apples, and found that he had three bushels left; what was his whole crop? how many bushels did he sell?

5. If you divide twenty pears among ten children, how many pears will each child receive? what part of the whole number of pears? What is one tenth of twenty? two tenths? five tenths? seven tenths?

What is one tenth of 70? *Answer:* 7.

6. What is one tenth of 20? 50? 90? 320?

7. What is three tenths of 40? of 70? of 80?

8. Find five tenths of 120; seven tenths of 30; nine tenths of 100.

9. Mr. Hale's three children and seven others went on a sleigh-ride; what part of the whole expense ought each child to pay?

10. Mr. Hale paid 60 cents for his children; how much did the sleigh-ride cost in all?

11. If 24 is four tenths of a number, what is one tenth of the number? what is ten tenths, or the number itself?

12. Mr. Blake received $14 for seven tenths of his

crop of apples; what did he receive for one tenth of the crop? for ten tenths, or the whole crop?

13. 14 is seven tenths of what number?

Answer: If 14 is seven tenths, one tenth is one seventh of 14, or 2, and ten tenths, or the whole number, is ten times 2, or 20. Therefore, 14 is seven tenths of 20.

14. 18 is three tenths of what number?
15. 30 is five tenths of what number?
16. 28 is seven tenths of what number?
17. 45 is nine tenths of what number?

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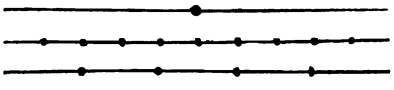
18. Here are three lines of equal length. The first is



divided into halves, the next is divided into tenths, and the last is divided into fifths. Measure and see how many tenths make a half, and how many tenths make one fifth.

19. How many cents are there in one tenth of a dollar? in one fifth of a dollar? in one fourth of a dollar? in one half of a dollar?

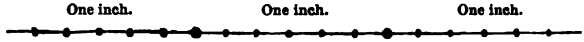
B. Problems.

1. How can you divide two apples among three boys?

Answer: First cut one of the apples into three equal parts, and give one of these parts, or thirds, to each boy; next do the same thing with the other apple. Each boy, then, will get one third of each of the two apples; now if the apples are alike, all their parts, or thirds, will be alike, and one third of two apples will amount to the same as two thirds of one apple.

2. If you divide two pints of milk among three boys, how much milk will you give to each?

3. What part of an inch is one sixth of three inches?

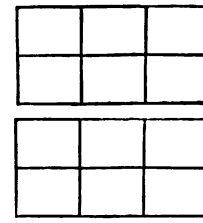


Explanation: This line is three inches long, and each inch has been divided into six equal parts; each part, then, is one sixth of an inch. In the whole line there are 3 times 6, or 18 parts; in one sixth of the line, then, there are one sixth of 18 or 3 parts (3 *sixths of an inch*). Therefore one sixth of 3 inches equals 3 sixths of one inch.

4. What part of an inch is one sixth of two inches?

5. If 4 boys, working together, earn 3 dollars, how many quarters of a dollar will be each boy's share?

6. How can you divide two loaves of cake equally among six boys?



You may cut each loaf into six equal parts, or sixths, and give each boy one piece of the first loaf, and one piece of the second loaf, or two pieces in all. If the loaves are alike, so that all the pieces are of the same size, you can, if you like, give a boy his two pieces from the same loaf. Each boy's share will be, then, two sixths of a loaf of cake.

7. If you had to divide two melons of the same size equally among six children, how much would you give to each?

In dividing two things equally among six persons you give to each two sixths of one of the things; what is, then, one sixth of two? *Answer:* Two sixths of one.

8. What is one eighth of five?

Answer: One eighth of five must be five times as great as one eighth of one; one eighth of five is, then, five eighths of one.

9. What is one fifth of four?

Answer: Four fifths of one.

10. What is one fourth of three? one fifth of three?

one sixth of four? one fourth of two? one seventh of five? one ninth of seven?

11. If you divide six oranges equally among eight persons, what part of an orange will each receive?

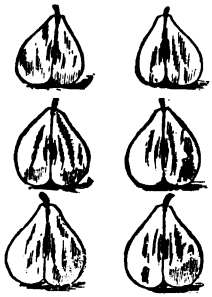
12. If a yard of cloth costs three dollars, what must you pay for a quarter of a yard?

13. How many acres are there in one twelfth of a ten-acre lot?

14. Farmer Brown's six-acre lot is divided into twelve equal parts by the ditches which drain the land. In one of these parts he wishes to raise potatoes; how many bushels will he get at the rate of 36 bushels to the acre?

15. How can you divide 3 pears among 2 boys?

Answer: I can give each boy a whole pear, and then divide the pear left over by giving each boy half a pear more. Each boy, then, will get *one pear and half a pear*, or, as it is sometimes written, *one and one half pears.*



Or, I can give each boy one half of each pear, or three half pears in all. Three half pears is usually written *three halves of a pear.*

Since three halves of a pear are equivalent to one pear and half of a pear, each boy will get the same amount, in whichever way the pears are divided.

16. How could you divide 8 apples equally among 6 boys?

Answer: I might divide each apple into six equal parts, and give each boy one piece (sixth) of each apple, or eight sixths in all.

Or, I might give each boy a whole apple, and then divide the two apples left over by giving each boy two sixths of an apple more.

Since eight sixths of an apple are equivalent to a whole apple and two sixths of an apple over, I give each boy the same quantity, whichever way I share the apples.

17. If I wish to divide 12 oranges equally among 9 girls, how much must I give to each?

Answer: Twelve ninths of an orange, or 1 orange and three ninths of an orange.

18. What is one eighth of twelve?

Answer: One eighth of twelve must be twelve times as great as one eighth of one; one eighth of twelve is equivalent, then, to twelve eighths of one, or to one and four eighths of one.

19. What is one sixth of fourteen?

Answer: Fourteen sixths of one, or two whole ones and two sixths.

20. What is one seventh of thirty?

Answer: Thirty sevenths of one, or four whole ones and two sevenths.

21. What is one eighth of twenty-six? one fifth of sixteen? one sixth of thirty-five? one ninth of forty-seven?

22. What is three sevenths of nine?

Answer: One seventh of nine is equal to nine sevenths of one; three sevenths of nine, then, must be three times nine sevenths of one, or twenty-seven sevenths of one, which is equivalent to three and six sevenths.

23. What is five eighths of six? three fourths of nine? two thirds of eight? five thirds of eight?

24. What is two sevenths of five? three fifths of eleven? six fifths of seven? nine eighths of four? five ninths of six?

C. Short Questions and Problems, reviewing the Principles of A and B.

1. If you cut an apple into two equal parts, what is one of those parts called?

2. How many halves of an apple will make the whole apple?

3. If you cut an apple into 3 equal parts, what is 1 of those parts called? what are 2 of the parts called?

4. How many thirds of an apple will make the whole apple?

5. If you cut an apple into 4 equal parts, what is 1 of those parts called? what are 2 of those parts called? what are 3 of them called?

6. How many fourths of an apple make the whole apple?

7. If an apple is cut into 5 equal parts, what is 1 of the parts called? what are 2 of the parts called? what 3? what 4?

8. How many fifths of an apple make the whole apple?

9. If an apple is cut into 6 equal parts, what is 1 of the parts called? what are 2 of the parts called? what 3? what 4? what 5?

10. How many sixths of an apple make the whole apple?

11. If an apple is cut into 7 equal parts, what is 1 of the parts called? what are 2 of the parts called? what 3? what 4? what 5? what 6?

12. How many half dollars in 1 dollar? in 2 dollars? in 3 dollars? in 3 dollars and a half?

13. A man had a bushel of corn, and wished to give 1 half of a bushel apiece to some laborers; how many could he give it to?

14. How many halves are there in 1?

15. A man divided 2 barrels of flour among his laborers, giving them 1 half of a barrel apiece; how many men did he give it to?

16. How many halves are there in 2?

17. In 3 bushels of corn how many half bushels?

18. How many halves are there in 3?

19. A boy divided 4 oranges among his companions, giving them 1 half of an orange apiece; how many boys did he give them to?

20. How many halves are there in 4?

21. A man having some laborers, gave them 1 half a dollar apiece; it took 3 dollars and 1 half a dollar to pay them; how many laborers were there?

22. How many halves are there in 3 and 1 half?

23. How many halves are there in 5?

24. How many halves are there in 7 and 1 half?

25. How many feet are there in 1 yard? in 2 yards? in 3 yards? in 3 and 2 thirds yards? What part of a yard is a foot?

How many thirds of a yard are there in 1 yard? in 2 yards? in 3 yards?

26. If you had 1 orange, and should divide it among your companions, giving them 1 third apiece, how many could you give it to?

27. How many thirds are there in 1?

28. If you cut 2 oranges each into 3 pieces, how many pieces would they make?

29. If you cut 3 oranges into 3 pieces each, how many pieces would they make?

30. If you cut 4 apples each into 3 pieces, how many pieces would they make?

31. How many thirds are there in 2? in 3? in 4? in 5?

32. If you had 2 bushels and 1 third of a bushel of corn to give to some poor persons, how many could you give it to if you should give them 1 third of a bushel apiece?

33. How many thirds are there in 2 and 1 third?

Answer: In 1 there are 3 thirds; therefore, in 2 there are 2 times 3 thirds, or 6 thirds; in 2 and 1 third, then, there are 6 thirds and 1 third, or 7 thirds.

34. If a horse can eat 1 third of a bushel of oats in 1 day, how many days would it take him to eat 3 bushels and 2 thirds of a bushel?

35. How many thirds are there in 3 and 2 thirds?

36. If 1 horse can eat 1 third of a bushel of oats in a day, how many horses will it take to eat 5 bushels and 2 thirds of a bushel in the same time?

37. In 5 and 2 thirds how many thirds?

38. In 7 and 1 third how many thirds?

39. If 1 horse will eat 1 fourth of a ton of hay in 1 month, how many horses will eat a ton in the same time?

How many will eat 2 tons? How many will eat 3 tons? 4 tons? 5 tons? 6 tons?

40. How many fourths are there in 1? in 2? in 3? in 4? in 5? in 6?

41. In 5 tons of hay and 3 fourths of a ton, how many fourths of a ton?

42. In 5 and 3 fourths how many fourths?

43. In 7 and 1 fourth how many fourths?

44. In 9 and 3 fourths how many fourths?

45. If a horse eats 1 fifth of a ton of hay in a month, how many horses will eat a ton in the same time? 2 tons? 3 tons? 4 tons? 5 tons?

46. How many fifths are there in 1? in 2? in 3? in 4? in 5? in 7? in 9?

47. How can you tell how many fifths there are in any number?

48. In 2 and 1 fifth how many fifths?

49. In 3 dollars and 2 fifths of a dollar how many fifths of a dollar?

50. In 3 and 2 fifths how many fifths?

51. In 5 and 3 fifths how many fifths?

52. In 6 and 4 fifths how many fifths?

53. How many sixths are there in 1? in 2? in 3? in 4? in 5? in 7? in 8?

54. In 2 and 2 sixths how many sixths?

55. In 3 and 4 sixths how many sixths?

56. In six and 5 sixths how many sixths?

57. How many sevenths are there in 1? in 2? in 3? in 4? in 6? in 9? in 10?

58. In 3 and 1 seventh how many sevenths?

59. In 5 and 3 sevenths how many sevenths?

60. In 7 and 5 sevenths how many sevenths?

61. How many eighths are there in 1? in 3? in 5? in 8?

62. In 2 and 3 eighths how many eighths?

63. In 3 and 5 eighths how many eighths?

64. In 5 and 7 eighths how many eighths?

65. How many ninths are there in 1? in 2? in 7? in 5?

66. In 2 and 2 ninths how many ninths?
67. In 4 and 3 ninths how many ninths?
68. In 6 and 4 ninths how many ninths?
69. In 8 and 7 ninths how many ninths?
70. How many tenths in 1? in 2? in 5? in 8?
71. In 3 and 3 tenths how many tenths?
72. In 4 and 7 tenths how many tenths?
73. In 8 and 9 tenths how many tenths?
74. In 7 and 4 tenths how many tenths?
75. In 9 and 8 tenths how many tenths?
76. In 7 and 4 sevenths how many sevenths?
77. In 9 and 2 thirds how many thirds?
78. In 10 and 3 fourths how many fourths?
79. In 8 and 4 fifths how many fifths?
80. In 7 and 5 ninths how many ninths?

D. Problems relating to the Reduction of Fractions greater than 1 to Whole Numbers and Fractions.

1. If 2 pints make a quart, what part of a quart is one pint? *Answer:* One half of a quart. What part of 2 is 1?

2. How many quarts are there in 3 pints?

Answer: 3 pints = 1 quart and 1 pint left over.
= 1 quart and one half of a quart.
= 1 and one half quarts.

Or, since a pint is one half of a quart, in 3 pints there are 3 halves of a quart, or 1 and 1 half quarts.

3. How many *twos* are there in 3?

Answer: 1 *two* and one half of *two*.

How many times two is 3?

Answer: 1 and one half times two.

4. How many quarts are there in 7 pints?

Answer: 3 quarts and one pint over; or, 3 and one half quarts.

How many times 2 is 7?

Answer: 3 and one half times 2.

5. If 3 feet make one yard, what part of a yard is one foot? 2 feet?

What part of 3 is 1? what part of 3 is 2?

6. How many yards are there in 5 feet?

Answer: 5 feet = 1 yard and 2 feet over.
= 1 yard and 2 thirds of a yard.
= 1 and 2 thirds yards.

How many threes are there in 5?

Answer: 1 three and 2 thirds of 3.

How many times 3 is 5?

Answer: 1 and 2 thirds times 3.

7. How many yards are there in 7 feet?

Answer: 2 and 1 third yards.

How many times 3 is 7?

Answer: 2 and 1 third times 3.

8. If 4 pecks make a bushel, how many bushels are there in 7 pecks? *Answer:* 1 and 3 quarters bushels.

How many times 4 is 7?

Answer: 1 and 3 fourths times 4.

9. If 8 quarts make a peck, how many pecks are there in 18 quarts? *Answer:* 2 and 2 eighths pecks.

How many times 8 is 18?

Answer: 2 and 2 eighths times 8.

10. How many pecks are there in 31 quarts?

Answer: 3 and 7 eighths pecks.

How many times 8 is 31?

Answer: 3 and 7 eighths times 8.

11. If 12 inches make a foot, how many feet are there in 29 inches? *Answer:* 2 and 5 twelfths feet.

How many times 12 is 29?

Answer: 2 and 5 twelfths times 12.

12. How many hours will it take you to travel 10 miles, if you travel 3 miles in an hour?

13. If there are 4 quarts in a gallon, how many gallons are there in 10 quarts?

14. How much broadcloth at $6 a yard can you buy for $17?

15. How many pounds of raisins at 18 cents a pound can you buy for 25 cents?

16. How many barrels of flour at $8 a barrel can you buy for $34?

17. In 27 quarts how many gallons?

18. If an orange is worth 6 apples, how many oranges can you buy for 40 apples?

19. If there are 4 pecks in a bushel, and you can buy a bushel of apples for $1, what is the price of a peck?

20. 4 halves are how many ones?

21. If you give 3 boys 1 half of an orange apiece, how many oranges will it take?

22. 3 halves are how many ones?

Answer: In 1 there are 2 halves; therefore
3 halves = 1 and 1 half left over.
= 1 and 1 half of 1.

23. If you give five men 1 half of a dollar apiece, how many dollars will it take?

24. 5 halves are how many ones?

25. 6 halves are how many ones?

26. 7 halves are how many ones?

27. A man divided some corn among 6 persons, giving them 1 third of a bushel apiece; how many bushels did it take?

28. 6 thirds are how many ones?

29. In 5 thirds there are how many ones?

30. A man gave eight paupers 1 third of a dollar apiece; how many dollars did it take?

Answer: Eight thirds, or 2 and 2 thirds dollars.

31. 8 thirds are how many ones?

32. 10 thirds are how many ones?

33. If a man spends 1 fourth of a dollar in one day, how many dollars will he spend in 8 days? how many in 7 days? how many in 11 days?

34. 8 fourths are how many ones?

35. 7 fourths are how many ones?

36. 11 fourths are how many ones?

37. 13 fourths are how many ones?

38. 18 fourths are how many ones?

39. If 1 fifth of a barrel of flour will last a family 1 day, how many barrels will last them 10 days? How many 8 days? 11 days? 17 days?

40. 10 fifths are how many ones?

41. 8 fifths are how many ones?
42. 11 fifths are how many ones?
43. 17 fifths are how many ones?
44. 18 sixths are how many ones?
45. 23 fifths are how many ones?
46. 21 sevenths are how many ones?
47. 24 eighths are how many ones?
48. 36 ninths are how many ones?
49. 30 tenths are how many ones?
50. 35 fourths are how many ones?
51. 37 eighths are how many ones?
52. 43 fifths are how many ones?
53. 48 ninths are how many ones?
54. 53 tenths are how many ones?
55. 57 eighths are how many ones?
56. 76 tenths are how many ones?
57. 78 ninths are how many ones?

E. Miscellaneous Questions and Problems.

1. Eleven are how many times 2?

Answer: 5 times 2 and 1 half of 2.

2. Twelve are how many times 2? 3? 4?
3. Fourteen are how many times 2? 4? 3?
4. If you had fifteen cents, how many cakes could you buy at 4 cents apiece? how many at 2 cents apiece? how many at 3 cents apiece? how many at 5 cents apiece?
5. Fifteen are how many times 4? 2? 3? 5?
6. Sixteen are how many times 5? 3? 6? 2? 7? 4?
7. Seventeen are how many times 6? 2? 7? 3? 5? 4?
8. Eighteen are how many times 4? 7? 9? 6? 3? 2? 5? 8?
9. Nineteen are how many times 3? 7? 4? 5? 8? 6? 9? 2? 10?
10. Twenty are how many times 6? 2? 8? 3? 9? 4? 10? 5? 7?

11. Twenty-one are how many times 7? 3? 8? 2? 4? 6? 9? 5? 10?

12. Twenty-two are how many times 3? 8? 5? 4? 9? 6? 7? 10? 2?

13. If you had 27 dollars, how much cloth could you buy at 9 dollars a yard? how much at 6 dollars a yard? how much at 4 dollars a yard? how much at 3 dollars a yard? how much at 7 dollars a yard? how much at 8 dollars a yard? how much at 5 dollars a yard? how much at 10 dollars a yard?

14. Twenty-seven are how many times 9? 6? 4? 3? 7? 8? 5? 10?

15. Twenty-four are how many times 6? 8? 7? 5? 2? 10? 3? 4? 9?

16. Twenty-nine are how many times 3? 7? 5? 9? 6? 8? 4? 10?

17. Twenty-three are how many times 4? 2? 7? 8? 3? 9? 6? 5? 10?

18. Mary bought 1 half of a yard of cloth for one dollar; what would be the price of a yard at the same rate?

19. If 1 half pint of cherries costs 2 cents, what will a pint cost?

20. If 1 fourth of a barrel of flour costs two dollars, what would a barrel cost?

21. 2 is 1 half of what number? 1 fourth of what number?

22. If 1 third of a yard of cloth costs 2 dollars, what will a yard cost?

23. 2 is 1 third of what number?

24. If 1 third of a yard of cloth costs 3 dollars, what will a yard cost?

25. 3 is 1 third of what number?

26. If 1 fourth of a firkin of butter costs 3 dollars, what will a firkin cost?

27. 3 is 1 fourth of what number?

28. A Western man bought 1 third of an acre of land for 4 dollars; what would an acre cost at that rate?

29. 4 is 1 third of what number?

30. If a man can ride 2 miles in 1 fifth of an hour, how far can he ride in an hour?

31. 2 is 1 fifth of what number?

32. A man, being asked the age of his eldest son, answered that his youngest son, who was 3 years old, was just 1 fifth of the age of his eldest son; how old was the eldest son?

33. 3 is 1 fifth of what number?

34. A man bought 1 sixth part of a hundred-weight of sugar for 2 dollars; what would a hundred-weight cost at the same rate?

35. 2 is 1 sixth of what number?

36. Bridget bought 1 fifth of a pound of starch for 3 cents; what was that a pound?

37. 5 is 1 fifth of what number?

38. Bought 1 fourth of a pound of cream of tartar for 12 cents; what was that a pound?

39. 12 is 1 fourth of what number?

40. A man can live at the Parker House, Boston, for $4 a day; what will it cost him, at this rate, to live there a week?

41. 4 is 1 seventh of what number?

42. 7 is 1 fifth of what number?

43. 5 is 1 third of what number?

44. 4 is 1 eighth of what number?

45. 6 is 1 sixth of what number?

46. 8 is 1 third of what number?

47. 9 is 1 fourth of what number?

48. 7 is 1 sixth of what number?

49. 8 is 1 seventh of what number?

50. 9 is 1 eighth of what number?

51. 8 is 1 tenth of what number?

52. 7 is 1 ninth of what number?

53. 6 is 1 fifth of what number?

54. 10 is 1 seventh of what number?

55. A man bought some cloth and some silk: for the silk he gave $4 per yard, which was twice as much as he gave for the cloth; what did he give per yard for the cloth?

56. 4 is 2 times what number?

57. If 2 thirds of a yard of cloth cost 6 dollars, what *would* 1 third cost?

58. 6 is 2 times what number?

59. If 3 fourths of a barrel of flour cost 6 dollars, what will 1 fourth of a barrel cost?

60. 6 is 3 times what number?

61. If 2 fifths of a pound of sausages cost 8 cents, what would 1 fifth of a pound cost?

62. 8 is 2 times what number?

63. If 3 fifths of a pound of grapes cost 9 cents, what will 1 fifth of a pound cost?

64. 9 is 3 times what number?

65. If 2 sevenths of a pound of spermaceti candles cost 10 cents, what will 1 seventh of a pound cost?

66. 10 is 2 times what number?

67. If 5 eighths of a pound of cotton cost 10 cents, what will 1 eighth cost?

68. 10 is 5 times what number?

69. If 2 thirds of a yard of cloth cost 4 dollars, what will 1 third cost? If 1 third of a yard costs 2 dollars, what will a yard cost?

70. If 4 is 2 thirds of some number, what is 1 third of the same number? 2 is 1 third of what number? Then 4 is 2 thirds of what?

71. If 2 thirds of a barrel of flour cost 6 dollars, what will 1 third of a barrel cost? If 1 third of a barrel costs 3 dollars, what will a barrel cost?

72. If 6 is two thirds of some number, what is 1 third of the same number? 3 is one third of what number? Then 6 is 2 thirds of what?

73. If 3 fourths of a barrel of beans cost $6, what will 1 fourth of a barrel cost? If 1 fourth of a barrel cost $2, what will a barrel cost?

74. If 6 is 3 fourths of some number, what is 1 fourth of the same number? 2 is 1 fourth of what number? Then 6 is 3 fourths of what?

75. If 2 fifths of a yard of fine lace cost $4, what will 1 fifth of a yard cost? If 1 fifth of a yard costs $2, what will a yard cost?

76. If 4 is 2 fifths of some number, what is 1 fifth of the same number? 2 is 1 fifth of what number? Then 4 is 2 fifths of what?

77. If 3 sevenths of a pound of cheese cost 6 cents, what will 1 seventh of a pound cost? If 1 seventh of a pound costs 2 cents, what will a pound cost?

78. If 6 is 3 sevenths of some number, what is 1 seventh of the same number? 2 is 1 seventh of what number? Then 6 is 3 sevenths of what?

79. If 2 sevenths of a barrel of fish cost 4 dollars, what will 1 seventh of a barrel cost? What will a barrel cost?

80. 4 is 2 sevenths of what number?

81. If 3 eighths of a pound of sausages cost 6 cents, what will 1 eighth of a pound cost? What will a pound cost?

82. 6 is 3 eighths of what number?

83. If eight cents will buy 2 fifths of a pound of saleratus, how many cents will buy a pound?

84. 8 is 2 fifths of what number?

85. A man bought 3 fourths of a hundred-weight of yellow ochre for 9 dollars; what was that a hundred weight?

86. 9 is 3 fourths of what number?

87. 8 is 4 ninths of what number?

88. 9 is 3 tenths of what number?

89. 10 is 5 sevenths of what number?

90. 12 is 3 fifths of what number?

91. 12 is 4 ninths of what number?

92. 10 is 2 sevenths of what number?

93. 14 is 7 fifths of what number?

94. 15 is 3 elevenths of what number?

95. 16 is 2 fifths of what number?

96. 18 is 6 tenths of what number?

97. 20 is 5 ninths of what number?

98. 21 is 3 ninths of what number?

99. 24 is 8 ninths of what number?

SECTION IX.

Fractions Continued.

A. Fractional Notation. Numerator and Denominator. Problems.

Halves, thirds, fourths, fifths, etc., of a thing are called FRACTIONS.

Fractions may be expressed by figures, thus:

One third by $\frac{1}{3}$.	Three fourths by $\frac{3}{4}$.
Two thirds by $\frac{2}{3}$.	Two fifths by $\frac{2}{5}$.
One fourth by $\frac{1}{4}$.	Four sevenths by $\frac{4}{7}$.

1. Read the fractions: $\frac{5}{7}$, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{1}{18}$ $\frac{3}{20}$, $\frac{7}{16}$, $\frac{6}{12}$, $\frac{5}{11}$, $\frac{1}{7}$, $\frac{4}{5}$, $\frac{7}{8}$.

2. Express one fifth by figures; three fifths; one sixth; four sixths; five sixths; three sevenths; one eighth; five eighths; one ninth; four ninths; one thirteenth; eleven thirteenths; one twenty-first; twenty twenty-firsts.

3. What is $\frac{2}{3}$ of 6? $\frac{3}{4}$ of 8? $\frac{3}{5}$ of 15?

4. What is $\frac{4}{7}$ of 14? $\frac{3}{8}$ of 32? $\frac{2}{3}$ of 18?

5. 14 is $\frac{2}{3}$ of what number?

6. 18 is $\frac{3}{4}$ of what number?

7. 25 is $\frac{5}{6}$ of what number?

8. If I plant $\frac{1}{8}$ of my garden with peas, $\frac{1}{8}$ with beans, $\frac{3}{8}$ with potatoes, and $\frac{1}{8}$ with tomatoes; how many eighths shall I have left for other vegetables?

How many eighths are $\frac{1}{8}+\frac{1}{8}+\frac{3}{8}+\frac{1}{8}$?

Add together $\frac{4}{5}$, $\frac{1}{5}$, and $\frac{3}{5}$.

9. How many sevenths are $\frac{1}{7}+\frac{3}{7}+\frac{2}{7}$?

10. How many ninths are $\frac{3}{9}+\frac{2}{9}+\frac{7}{9}$?

As we have seen, it requires two numbers to express a fraction; one (which we write below the line) to show into how many equal parts the thing which we are talking about is divided; the other to show how many of these parts are taken.

For instance, $\frac{3}{7}$ of an apple is that portion of an apple which we get by cutting the apple into 7 equal parts, and then taking 3 of these parts. $\frac{5}{12}$ of an orange represents that portion of an orange which we could get by cutting the orange into 12 equal parts, and taking 5 of these parts.

11. What is meant by $\frac{3}{5}$ of an orange? $\frac{1}{8}$ of an apple? $\frac{7}{12}$ of a melon?

The number which shows into how many parts the thing is to be divided is always written *below the line*, and is called the DENOMINATOR of the fraction. The number which shows how many of these equal parts are to be taken is written *above the line*, and is called the NUMERATOR. Thus, in $\frac{5}{9}$, 9 is the *denominator* and 5 is the *numerator*.

12. In the fraction $\frac{7}{8}$, what does the 8 denote? the 7? which is the numerator? which the denominator?

Which is the numerator and which is the denominator in each of the following fractions? $\frac{4}{11}$, $\frac{5}{7}$, $\frac{3}{10}$, $\frac{2}{3}$, $\frac{12}{13}$.

Notice that when a thing is divided into 3 equal parts, the parts are called *thirds;* when a thing is divided into 4 equal parts, the parts are called *fourths;* when a thing is divided into 19 equal parts, the parts are called *nineteenths;* that is, the fraction takes its name from the *denominator*.

13. Read $\frac{1}{24}$, $\frac{3}{36}$, $\frac{6}{43}$, $\frac{7}{56}$, $\frac{37}{41}$, $\frac{113}{41}$, $\frac{384}{1816}$.

14. Mary had $\frac{3}{4}$ of a melon, but gave away $\frac{1}{4}$; how how much had she left?

What is $\frac{3}{4}-\frac{1}{4}$? $\frac{8}{9}-\frac{4}{9}$? $\frac{12}{17}-\frac{7}{17}$? $\frac{9}{21}-\frac{9}{21}$?

15. Mr. Wood bought a yacht, and sold $\frac{1}{5}$ of it to Mr. Shellabarger; what portion of the yacht did Mr. Wood then own?

16. How much is $\frac{1}{8}+\frac{6}{8}-\frac{3}{8}$? $\frac{5}{16}-\frac{2}{16}+\frac{7}{16}$?

$\frac{9}{32}+\frac{9}{32}-\frac{5}{32}+\frac{11}{32}$? $\frac{12}{49}+\frac{7}{49}+\frac{6}{49}-\frac{11}{49}$?

17. At two cents a yard, what will 3 yards and $\frac{1}{2}$ of a yard of tape cost?

Answer: 3 yards will cost 3 times 2 cents or 6 cents; $\frac{1}{2}$ of a yard will cost $\frac{1}{2}$ of 2 cents or 1 cent: therefore 3 yards and $\frac{1}{2}$ of a yard will cost 6 cents + 1 cent, or 7 cents.

3 times $2 + \frac{1}{2}$ of 2 are how many?

18. At three dollars a yard, what will 4 yards and $\frac{1}{3}$ of a yard of cloth cost?

4 times $3 + \frac{1}{3}$ of 3 are how many?

19. At 3 dollars a barrel, what will 3 barrels and $\frac{2}{3}$ of a barrel of turnips cost?

20. 3 times $3 + \frac{2}{3}$ of 3 are how many?

21. If a man earns 4 dollars in a week, how many dollars will he earn in 3 weeks and $\frac{1}{4}$ of a week?

22. 3 times $4 + \frac{1}{4}$ of 4 are how many?

23. If a yard of cloth costs 4 dollars, what will 5 yards and $\frac{3}{4}$ of a yard cost?

24. 5 times $4 + \frac{3}{4}$ of 4 are how many?

25. If a man spends five dollars in a week, how many dollars will he spend in 3 weeks and $\frac{1}{7}$ of a week? How much in 5 weeks and $\frac{2}{7}$ of a week?

26. 3 times $5 + \frac{1}{5}$ of 5 are how many?

27. 5 times $5 + \frac{2}{5}$ of 5 are how many?

28. 6 times $5 + \frac{3}{5}$ of 5 are how many?

29. If beans are worth six dollars a barrel, what would 4 barrels and 1 sixth of a barrel cost? How much would 7 barrels and 5 sixths of a barrel cost?

30. 4 times 6 + 1 sixth of 6 are how many?

31. 7 times 6 + 5 sixths of 6 are how many?

32. At 7 dollars a barrel, what will 3 barrels and 1 seventh of a barrel of flour cost? What will 5 barrels and 2 sevenths of a barrel cost?

33. 3 times 7 + 1 seventh of 7 are how many?

34. 5 times 7 + 2 sevenths of 7 are how many?

35. 8 times 5 + 4 fifths of 5 are how many?

36. 8 times 6 + 3 sixths of 6 are how many?

37. At 8 dollars a yard, what will 4 yards and 1 eighth of a yard of broadcloth cost?

38. 4 times 8 + 1 eighth of 8 are how many?

39. 2 times 7 + 3 sevenths of 7 are how many?

40. 8 times 7 + 4 sevenths of 7 are how many?
41. 9 times 7 + 6 sevenths of 7 are how many?
42. 3 times 8 + 5 eighths of 8 are how many?
43. 9 times 8 + 7 eighths of 8 are how many?
44. If a hundred-weight of sugar costs 9 dollars, what will 2 hundred-weight and 1 ninth of a hundred-weight cost? What will 5 hundred-weight and 2 ninths of a hundred-weight cost?
45. 2 times 9 + $\frac{1}{9}$ of 9 are how many?
46. 5 times 9 + 2 ninths of 9 are how many?
47. 6 times 9 + $\frac{4}{9}$ of 9 are how many?
48. 2 times 10 + 3 tenths of 10 are how many?
49. 7 times 9 + $\frac{7}{9}$ of 9 are how many?
50. 5 times 10 + 4 tenths of 10 are how many?
51. 8 times 9 + $\frac{8}{9}$ of 9 are how many?
52. 4 times 10 + 7 tenths of 10 are how many?
53. 6 times 10 + $\frac{9}{10}$ of 10 are how many?

54. If a breakfast for 1 man cost 1 third of a dollar, what would a breakfast for 2 men cost?
55. How much is 2 times 1 third?
56. If it takes you 1 third of an hour to travel 1 mile, how long will it take you to travel 3 miles?
57. How much is 3 times 1 third?
58. If 1 man can eat 1 third of a pound of meat at a meal, how much can 5 men eat?
59. How much is 7 times 1 third?
60. If 1 man can eat 2 thirds of a pound of meat for dinner, how many thirds of a pound would 3 men eat?
61. How much is 2 times 2 thirds?
62. A man gave to 4 paupers 2 thirds of a dollar apiece, how many thirds of a dollar did he give them all? how many dollars?
63. 5 times 2 thirds are how many thirds? how many times 1?
64. If you give 3 men 1 fourth of a dollar apiece, how many fourths of a dollar will it take?
65. 3 times 1 fourth are how many fourths?
66. If you give 3 men 3 fourths of a bushel of corn

apiece, how many fourths of a bushel will it take? how many bushels?

67. 5 times 3 fourths are how many fourths? how many times 1?

68. If 1 horse eats 1 fifth of a bushel of oats in a day, how much will 4 horses eat in the same time?

69. 3 times 1 fifth are how many fifths?

70. If 1 man can earn 3 fifths of a dollar in a day, how much can he earn in 4 days?

71. 7 times 3 fifths are how many fifths? how many times 1?

72. If a family consumes 2 sevenths of a barrel of flour in a week, how much would they consume in 5 weeks?

73. 6 times 2 sevenths are how many sevenths? how many times 1?

74. 5 times 3 eighths are how many eighths? how many times 1?

75. How much is 6 times 3 fifths? $7 \times \frac{4}{8}$? $5 \times \frac{4}{3}$? $6 \times \frac{8}{3}$? $7 \times \frac{8}{10}$? $5 \times \frac{7}{10}$?

76. How much is 6 times 7 eighths? $9 \times \frac{5}{8}$? $8 \times \frac{5}{7}$? $7 \times \frac{5}{6}$? $8 \times \frac{7}{4}$? $7 \times \frac{4}{5}$? $5 \times \frac{3}{8}$?

B. "Proper" and "Improper" Fractions. "Mixed Numbers."

Fractions like $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{5}{8}$, in which the numerator is smaller than the denominator, are called PROPER fractions.

Fractions like $\frac{3}{2}$, $\frac{4}{3}$, $\frac{7}{4}$, $\frac{8}{5}$, in which the numerator is larger than the denominator, are called IMPROPER fractions.

1. Which of the following are proper fractions and which improper fractions? $\frac{1}{7}$, $\frac{5}{2}$, $\frac{7}{8}$, $\frac{7}{6}$, $\frac{7}{5}$, $\frac{9}{10}$, $\frac{11}{10}$, $\frac{10}{11}$.

2. Change $\frac{3}{2}$ to a whole number and a fraction.

Answer: $\frac{3}{2} = \frac{2}{2} + \frac{1}{2} = 1 + \frac{1}{2}$ and this is usually written $1\frac{1}{2}$.

3. Change $\frac{11}{4}$ to a whole number and a fraction.

Answer: 2 and $\frac{3}{4}$; usually written $2\frac{3}{4}$.

4. A whole number and a fraction like $1\frac{1}{2}$, $2\frac{3}{4}$, $4\frac{1}{3}$, is called a MIXED NUMBER.

Change the mixed number $4\frac{1}{3}$ to an improper fraction.

Answer: $4\frac{1}{3} = 4 + \frac{1}{3} = \frac{12}{3} + \frac{1}{3} = \frac{13}{3}$.

5. Change the mixed numbers $3\frac{1}{2}$, $1\frac{3}{4}$, $2\frac{1}{3}$, $4\frac{1}{4}$, $3\frac{2}{3}$ to improper fractions.

6. Change the improper fractions $\frac{4}{2}$, $\frac{4}{3}$, $\frac{7}{4}$, $\frac{8}{5}$, $\frac{13}{8}$, $\frac{8}{7}$ to mixed numbers.

7. If 1 bushel of wheat costs $1\frac{1}{2}$ dollars what will 2 bushels cost? *Answer:* 3 dollars.

8. How much is 2 times $1\frac{1}{2}$?

Solution: 2 times 1 are 2, and 2 times 1 half are 2 halves or 1, which, added to 2, makes 3.

Or: $1\frac{1}{2} = \frac{3}{2}$; therefore 2 times $1\frac{1}{2} = 2 \times \frac{3}{2} = \frac{6}{2} = 3$.

9. If a barrel of apples costs $2\frac{1}{2}$ dollars, what will 3 barrels cost?

10. How much is 3 times $2\frac{1}{2}$?

11. If a barrel of sweet potatoes costs $3\frac{1}{2}$ dollars, what will 2 barrels cost?

12. How much is 5 times $3\frac{1}{2}$? $6 \times 3\frac{1}{2}$?

13. If a box of butter costs $2\frac{1}{3}$ dollars, what will 3 boxes cost?

14. How much is 4 times $2\frac{1}{3}$?

15. If you give to two persons 3 bushels and a third of a bushel of wheat apiece, how many bushels will it take?

16. How much is 5 times $3\frac{1}{3}$?

17. If you give to 4 persons each 2 oranges and 1 fourth of an orange, how many oranges will it take?

18. How much is 5 times $2\frac{1}{4}$?

19. If it takes 3 yards and 2 thirds of a yard of cloth to make a suit of clothes, how many yards will it take to make 2 suits?

20. How much is 4 times $3\frac{2}{3}$?

21. If a family consumes 2 bushels and 2 thirds of a bushel of potatoes in 1 month, how much will they consume in 3 months?

22. How much is 5 times $2\frac{2}{3}$? $4 \times 3\frac{3}{4}$? $2 \times 3\frac{1}{4}$? $3 \times 3\frac{3}{4}$? $3 \times 5\frac{1}{4}$?

23. If a horse eats 3 tons and 1 fifth of a ton of hay in a year, how much will 2 horses eat in the same time?

24. How much is 4 times $3\frac{1}{8}$?

25. If a man can travel 4 miles and 2 fifths of a mile in one hour, how far can he travel in 3 hours?

26. How much is 5 times $4\frac{2}{3}$? $3 \times 5\frac{3}{5}$? $4 \times 6\frac{2}{3}$? $3 \times 2\frac{1}{8}$? $2 \times 5\frac{3}{8}$?

27. If a yard of cloth costs 4 dollars and 5 sixths of a dollar, what will 4 yards cost?

28. How much is 7 times $4\frac{5}{8}$? $2 \times 3\frac{3}{7}$? $3 \times 4\frac{3}{7}$?

29. If a barrel of flour costs $7\frac{3}{8}$ dollars, what will 5 barrels cost at that rate?

30. How much is 3 times $4\frac{5}{8}$? $4 \times 3\frac{7}{8}$?

31. A man bought 8 yards of cloth at 9 dollars and 3 tenths a yard; how much did it come to?

32. How much is 6 times $2\frac{3}{8}$? $4 \times 5\frac{3}{8}$?

33. A man bought 10 barrels of pears at 3 and 5 sixths dollars a barrel; how much did he pay?

34. How much is 3 times $7\frac{2}{10}$?

35. A boy wished to give 5 other boys 1 half pint of chestnuts apiece; how many pints would it take?

36. A boy wished to give 3 other boys 3 fourths of an orange apiece; how many oranges would it take?

37. A man gave to 10 persons 1 fifth of a bushel of corn apiece; how many bushels did it take?

38. A man gave to 7 men 3 fourths of a gallon of milk apiece; how many gallons of milk did it take?

39. If it takes $1\frac{1}{4}$ yards of cloth to make a pair of pantaloons, how many yards would it take to make 8 pairs?

40. If a family consumes $2\frac{3}{8}$ bushels of grain in 1 week, how many bushels would they consume in 4 weeks?

41. If a horse eats $3\frac{1}{7}$ bushels of oats in 1 week, how many bushels would he eat in 8 weeks?

42. If a horse eats $5\frac{3}{8}$ loads of hay in 1 year, how many loads would 6 horses eat?

43. If a man travels $4\frac{5}{8}$ miles in an hour, how many miles would he travel in 8 hours?

44. If in an orchard of 10 trees each tree bears $8\frac{3}{7}$ bushels, how many bushels will the whole orchard bear?

45. If a man can build $5\frac{1}{8}$ rods of wall in 1 day, how many rods can he build in 8 days?

46. If 3 men can build a piece of wall in $4\frac{3}{5}$ days, how many days would it take 1 man to build it?

47. If 1 man can build $7\frac{3}{7}$ rods of wall in a day, how many rods would 10 men build?

48. If 1 man can build $3\frac{1}{2}$ rods of wall in 1 day, how many rods would 3 men build in 4 days?

49. If it takes $1\frac{3}{7}$ yards of cloth to make 1 pair of pantaloons, and $2\frac{1}{7}$ yards for a coat; how many yards would it take to make 3 pairs of pantaloons and 3 coats.

50. A man bought 1 half of a yard of cloth for 1 dollar and 1 half; what was that a yard?

51. $1\frac{1}{2}$ is the half of what number?

52. If $\frac{1}{3}$ of a yard of cloth costs 1 dollar and $\frac{3}{4}$ of a dollar, how much does a yard cost?

Answer: 1 dollar and $\frac{3}{4}$ of a dollar is $\frac{7}{4}$ of a dollar. If $\frac{1}{3}$ of a yard costs $\frac{7}{4}$ of a dollar, $\frac{3}{3}$ or a whole yard will cost 3 times $\frac{7}{4}$ of a dollar, or $\frac{21}{4}$ of a dollar, which is $5\frac{1}{4}$ dollars.

Or: If $\frac{1}{3}$ of a yard costs $1\frac{3}{4}$ dollars, $\frac{3}{3}$ or a whole yard will cost 3 times $1\frac{3}{4}$ dollars. 3 times 1 dollar is 3 dollars and 3 times $\frac{3}{4}$ of a dollar is $\frac{9}{4}$, or $2\frac{1}{4}$ dollars. Therefore a yard will cost $3+2\frac{1}{4}$, or $5\frac{1}{4}$ dollars.

53. $2\frac{2}{3}$ is $\frac{1}{3}$ of what number?

54. If $\frac{1}{2}$ of a barrel of beans costs $2\frac{1}{4}$ dollars, how much will a barrel cost?

55. $2\frac{1}{2}$ is $\frac{1}{2}$ of what number?

56. If $\frac{1}{4}$ of a box of lemons costs 3 dollars and $\frac{3}{5}$ of a dollar, what will a box cost?

57. $2\frac{2}{5}$ is $\frac{1}{4}$ of what number?

58. $3\frac{1}{2}$ is $\frac{1}{4}$ of what number?

59. If $\frac{1}{3}$ of a barrel of pork costs $4\frac{1}{3}$ dollars, what will a barrel cost?

60. $4\frac{2}{3}$ is $\frac{1}{3}$ of what number?

61. If $\frac{1}{8}$ of a barrel of fish costs 2 dollars and $\frac{1}{8}$ of a dollar, what will a barrel cost?

62. $3\frac{2}{5}$ is $\frac{1}{5}$ of what number?

63. If $\frac{1}{5}$ of a barrel of salmon costs $3\frac{2}{7}$ dollars, what is that a barrel?

64. $4\frac{3}{7}$ is $\frac{1}{5}$ of what number?

65. If a man can travel 4 miles and $\frac{3}{7}$ of a mile in $\frac{1}{5}$ of a day, how far can he travel in a whole day?

66. $5\frac{3}{7}$ is $\frac{1}{5}$ of what number?

67. $2\frac{4}{5}$ is $\frac{1}{7}$ of what number?

68. $6\frac{3}{8}$ is $\frac{1}{5}$ of what number?

69. $7\frac{5}{8}$ is $\frac{1}{5}$ of what number?

70. $8\frac{3}{7}$ is $\frac{1}{4}$ of what number?

71. $5\frac{3}{10}$ is $\frac{1}{7}$ of what number?

72. $8\frac{2}{8}$ is $\frac{1}{5}$ of what number?

73. $9\frac{4}{5}$ is $\frac{1}{8}$ of what number?

74. $6\frac{4}{10}$ is $\frac{1}{10}$ of what number?

75. $7\frac{4}{5}$ is $\frac{1}{5}$ of what number?

76. $8\frac{7}{10}$ is $\frac{1}{7}$ of what number?

C. Problems relating to Improper Fractions.

1. If 2 thirds of a barrel of sweet potatoes costs 3 dollars, what does 1 third of a barrel cost?

2. 3 is 2 times what number?

Answer: 3 is 2 times the half of 3; but 1 half of 3 is 3 halves, or $1\frac{1}{2}$; therefore 3 is 2 times $1\frac{1}{2}$.

3. If $\frac{3}{5}$ of a yard of cloth costs 4 dollars, what will $\frac{1}{5}$ of a yard cost?

4. 5 is 3 times what number?

5. If 3 sevenths of a barrel of pork costs 2 dollars, what will 1 seventh of a barrel cost?

6. 2 is 3 times what number?

7. If 4 thirds of a bunch of shingles costs 5 dollars, what does 1 third of a bunch cost?

8. 7 is 4 times what number?

9. 2 is 4 times what number?

10. A man bought $\frac{4}{5}$ of a barrel of flour for 3 dollars; what would be the price of $\frac{1}{5}$ of a barrel at the same rate?

11. 5 is 4 times what number?

12. A man bought $\frac{4}{5}$ of a hundred-weight of sugar for 6 dollars; what would $\frac{1}{5}$ of a hundred-weight cost at the same rate?

13. 7 is 5 times what number?

14. 3 is 5 times what number?

15. 8 is 5 times what number?

16. 9 is 4 times what number?

17. 11 is 6 times what number?

18. 13 is 7 times what number?

19. 14 is 8 times what number?

20. 17 is 5 times what number?

21. 18 is 8 times what number?

22. 17 is 9 times what number?

23. 15 is 10 times what number?

24. 20 is 9 times what number?

25. 22 is 10 times what number?

26. 24 is 7 times what number?

27. If $\frac{2}{3}$ of a barrel of sweet potatoes costs 3 dollars, what will $\frac{1}{3}$ of a barrel cost? What will the whole barrel cost?

28. If 5 is 2 thirds of some number, what is 1 third of the same number? $2\frac{1}{2}$ is 1 third of what number? Then 5 is two thirds of what number?

29. If 3 fourths of a barrel of flour costs 5 dollars, what will 1 fourth of a barrel cost? What will the whole barrel cost?

30. If 8 is 3 fourths of some number, what is 1 fourth of the same number? $2\frac{2}{3}$ is 1 fourth of what number? Then 8 is 3 fourths of what number?

31. A man bought $\frac{3}{7}$ of a barrel of oil for 5 dollars; how much will $\frac{1}{7}$ cost at the same rate? how much would a barrel cost?

32. If 9 is $\frac{3}{7}$ of some number, what is $\frac{1}{7}$ of the same number? $4\frac{1}{2}$ is $\frac{1}{7}$ of what number? Then 9 is $\frac{3}{7}$ of what number?

33. If 12 is $\frac{4}{5}$ of some number, what is $\frac{1}{5}$ of the same number? $2\frac{2}{5}$ is $\frac{1}{5}$ of what number? Then 12 is $\frac{4}{5}$ of what number?

34. If a man can do $\frac{7}{9}$ of a piece of work in 4 days, how long would it take him to do $\frac{1}{9}$ of it? how long would it take him to do the whole?

35. If 3 is $\frac{7}{9}$ of some number, what is $\frac{1}{9}$ of the same number? $\frac{3}{7}$ is $\frac{1}{9}$ of what number? Then 3 is $\frac{7}{9}$ of what number?

36. If $\frac{3}{8}$ of a ton of Cannel coal cost 8 dollars, what is the whole ton worth?

37. 7 is $\frac{3}{8}$ of what number?

38. A man bought $\frac{6}{7}$ of a cask of raisins for 5 dollars; what was the whole cask worth?

39. 8 is $\frac{6}{7}$ of what number?

40. A man had $\frac{4}{5}$ of a week's board for 3 dollars; how much is that for a whole week?

41. 3 is $\frac{4}{5}$ of what number?

42. 9 is $\frac{4}{5}$ of what number?

43. 10 is $\frac{7}{8}$ of what number?

44. 11 is $\frac{8}{9}$ of what number?

45. 12 is $\frac{7}{8}$ of what number?

46. 15 is $\frac{3}{7}$ of what number?

47. 17 is $\frac{3}{10}$ of what number?

48. A man bought $\frac{5}{8}$ of a lot of land for 19 dollars; what would the whole lot cost?

49. 19 is $\frac{3}{8}$ of what number?

50. 21 is $\frac{4}{5}$ of what number?

51. A man bought $\frac{5}{8}$ of a ton of logwood for 23 dollars; what would a ton cost at that rate?

52. 23 is $\frac{7}{8}$ of what number?

53. 24 is $\frac{3}{10}$ of what number?

54. 29 is $\frac{4}{7}$ of what number?

55. 31 is $\frac{5}{8}$ of what number?

56. 33 is $\frac{6}{7}$ of what number?

57. 38 is $\frac{4}{9}$ of what number?

58. A man bought 1 acre and 1 seventh of an acre of land for 41 dollars; what was that an acre?

NOTE. — *1 acre and 1 seventh of an acre is the same as 8 sevenths of an acre. If 8 sevenths of an acre cost 41 dollars, what does an acre cost?*

59. 35 is 8 sevenths of what number?

60. A man bought 1 ton and $\frac{4}{5}$ of a ton (that is, $\frac{9}{5}$ of a ton) of fustic for 43 dollars; what was that a ton?

61. 52 is $\frac{8}{9}$ of what number?

62. I bought 2 tons and $\frac{3}{7}$ of a ton of logwood for 48 dollars; what was that a ton?

63. 67 is $\frac{9}{7}$ of what number?

64. 53 is $\frac{7}{10}$ of what number?

65. 58 is $\frac{8}{8}$ of what number?

66. 61 is $\frac{10}{9}$ of what number?

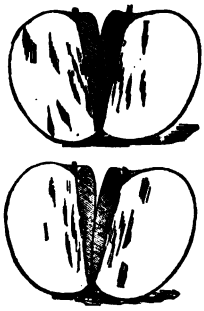
67. I bought 2 barrels and $\frac{1}{5}$ of a barrel of oil (that is, $\frac{11}{5}$ of a barrel) for 65 dollars; what was that a barrel?

68. 65 is $\frac{11}{5}$ of what number?

69. 71 is $\frac{8}{11}$ of what number?

D. Common Denominator. Problems. Illustrations.

1. If you divide an apple into two equal parts, and then divide each one of these into two equal parts, how many pieces will you have? What part of the whole apple will each piece be?



Answer: There will be 4 equal pieces, so that each piece will be one fourth of the apple.

$\frac{1}{2}$ of an apple is equal to how many fourths of an apple?

2. How many fourths of an orange are there in $\frac{1}{2}$ of an orange?

Answer: 2 fourths of an orange.

3. If you give $\frac{1}{2}$ of an orange to one boy and $\frac{1}{4}$ to another, how much more do you give the first than the second?

4. If you give $\frac{1}{2}$ of an orange to one boy and $\frac{1}{4}$ to another, how many times $\frac{1}{4}$ of an orange do you give away? how many fourths of the orange do you have left?

How many times $\frac{1}{4}$ are $\frac{1}{2}$ and $\frac{1}{4}$?

5. A man gave to one laborer $\frac{1}{4}$ of a bushel of wheat, and to another $\frac{3}{4}$ of a bushel; how many times $\frac{1}{4}$ of a bushel did he give to both? how many bushels?

How many times $\frac{1}{4}$ are $\frac{1}{2}$ and $\frac{3}{4}$?

6. If you divide an inch into halves, and then divide each of these halves into 3 equal parts, how many parts will you have? what portion of an inch will each part be?

How many sixths of an inch are there in $\frac{1}{2}$ of an inch?

7. A man gave $\frac{1}{2}$ of a barrel of flour to one person, and $\frac{3}{8}$ of a barrel to another; to which did he give the more?

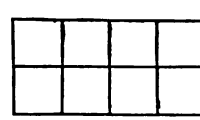
8. A man, paying some money to his laborers, gave each man $\frac{1}{2}$ of a dollar, and each boy $\frac{1}{8}$ of a dollar; how much more did he give to a man than to a boy? how much did a man and a boy receive?

9. What is the difference between $\frac{1}{2}$ and $\frac{1}{8}$? what is the sum of $\frac{1}{2}$ and $\frac{1}{8}$?

10. If a man earns $\frac{5}{8}$ of a dollar in a day, and a boy $\frac{1}{2}$ of a dollar, how much more does the man earn than the boy?

11. What is the difference between $\frac{4}{8}$ and $\frac{1}{2}$?

12. If you cut a loaf of cake into two equal parts, and then cut each part into quarters, how many pieces will you have in all? what part of the whole loaf will each piece be?



13. How many eighths of a loaf are there in $\frac{1}{2}$ of a loaf?

In $\frac{1}{2}$ of anything there are how many eighths?

14. If you cut a line or loaf of cake into quarters, and then cut each one of these quarters into thirds, how many pieces will you have in all? what part of the whole will each piece be? into how many twelfths of a cake can you cut one fourth of a cake?

How many twelfths are there in $\frac{1}{4}$?

15. Into how many eighths of an apple can you cut $\frac{1}{4}$ of an apple?

How many eighths are there in $\frac{1}{4}$?

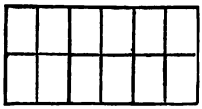
16. If you cut one quarter of a sheet of paper into 4 equal parts, what portion of the whole sheet will each part be?

[The pupil should find out the answer by actually cutting a sheet of paper.]

If one cake of chocolate will make 16 cups, how many cups will $\frac{1}{4}$ of a cake make?

How many sixteenths are there in $\frac{1}{4}$?

17. This figure is supposed to represent a sheet of paper divided into equal pieces. What part of the whole sheet is one of these pieces?



2 pieces form what part of the whole sheet? How many twelfths are there in one sixth?

3 pieces form what part of the whole sheet? 9 pieces? How many twelfths are there in one fourth? how many in 3 fourths?

4 pieces form what part of the whole sheet? 8 pieces? How many twelfths are there in one third? how many in 2 thirds?

6 pieces form what part of the whole sheet? How many twelfths are there in one half?

18. Take a sheet of paper 6 inches long and 4 inches wide, and draw a line on it so as to divide it into halves; now draw a second line so as to divide the sheet into quarters; now draw 4 other lines, so as to divide the sheet into twelfths.

Cut out with the scissors one twelfth of the sheet; also one sixth. How long and how wide are each of these pieces?

Cut out one fourth of the sheet; also one third. How long and how wide are each of these pieces?

How many twelfths of the sheet have you cut out in all? How many twelfths, then, are $\frac{1}{12}+\frac{1}{6}+\frac{1}{4}+\frac{1}{3}$?

19. How many twelfths are equal to $\frac{1}{3}$?

We may ask ourselves into how many twelfths of anything (for instance, a sheet of paper or a cake) we can cut $\frac{1}{3}$ of the thing, or we may reason thus:

In anything there are 12 twelfths of that thing; in $\frac{1}{3}$ of the thing, then, there must be $\frac{1}{3}$ of 12 twelfths, or 4 twelfths. Therefore $\frac{1}{3} = \frac{4}{12}$.

20. $\frac{1}{2}$ is equal to how many tenths?

Answer: In 1 there are 10 tenths; in $\frac{1}{2}$ of 1, then, there must be $\frac{1}{2}$ of 10 tenths, or 5 tenths. Therefore $\frac{1}{2} = \frac{5}{10}$.

21. $\frac{1}{5}$ is equal to how many thirtieths?

Answer: In 1 there are 30 thirtieths; in $\frac{1}{5}$ of 1, then, there must be $\frac{1}{5}$ of 30 thirtieths, or 6 thirtieths. Therefore $\frac{1}{5} = \frac{6}{30}$.

22. $\frac{1}{2}$ is equal to how many twelfths? fourteenths? twenty-fourths?

23. $\frac{1}{3}$ is equal to how many ninths? fifteenths? twenty-fourths?

24. $\frac{1}{4}$ is equal to how many twentieths? twenty-fourths? thirty-seconds?

25. $\frac{1}{7}$ is equal to how many fourteenths? twenty-eighths?

26. How many thirty-sixths does it take to make $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{6}$? $\frac{1}{9}$? $\frac{1}{12}$? $\frac{1}{18}$?

27. How many sixtieths does it take to make $\frac{1}{15}$? $\frac{1}{12}$? $\frac{1}{10}$? $\frac{1}{6}$? $\frac{1}{5}$? $\frac{1}{4}$? $\frac{1}{3}$? $\frac{1}{2}$?

28. A man bought $\frac{1}{3}$ of a bushel of wheat at one time, and $\frac{2}{6}$ of a bushel at another; at which time did he buy the more?

29. An ounce of Mary's medicine makes 48 doses; what part of an ounce is there in a dose? how many doses are there in a quarter of an ounce? $\frac{1}{4}$ is equal to how many forty-eighths?

30. $\frac{2}{3}$ are equal to how many ninths?

Answer: One third equals 3 ninths, therefore two thirds must equal 2 times 3 ninths or 6 ninths. $\frac{2}{3} = \frac{6}{9}$.

31. $\frac{4}{7}$ are equal to how many twenty-eighths?

Answer: One seventh equals 4 twenty-eighths, therefore *five* sevenths must equal 5 times 4 twenty-eighths or 20 twenty-eighths.

32. Show that $\frac{5}{9} = \frac{20}{36}$; $\frac{3}{8} = \frac{21}{56}$; $\frac{2}{11} = \frac{6}{33}$; $\frac{5}{6} = \frac{15}{18}$.

33. $\frac{5}{8}$ are how many times $\frac{1}{18}$?

$\frac{4}{7}$ are how many times $\frac{1}{21}$?
$\frac{5}{7}$ are how many times $\frac{1}{35}$?

34. $\frac{3}{5}$ are how many times $\frac{1}{56}$?
$\frac{3}{5}$ are how many times $\frac{1}{35}$?
$\frac{3}{10}$ are how many times $\frac{1}{20}$?

35. How many twelfths are there in $\frac{2}{3}$? in $\frac{3}{4}$? in $\frac{5}{6}$?

36. How many eighteenths are there in $\frac{2}{3}$? in $\frac{3}{6}$? in $\frac{2}{6}$?

37. How many forty-eighths are there in $\frac{3}{4}$? in $\frac{3}{8}$? in $\frac{7}{8}$? in $\frac{5}{12}$? in $\frac{7}{16}$? in $1\frac{1}{4}$?

38. A man bought $\frac{2}{3}$ of a yard of cloth at one time, and $\frac{3}{6}$ of a yard at another; how many sixths of a yard did he buy altogether? at which time did he buy the more?

39. What shall we get if we multiply both numerator and denominator of $\frac{1}{2}$ by 2?

First: If we multiply the denominator by 2 we get

$$\frac{1}{2\times 2}=\frac{1}{4}.$$

Second: If we now multiply the numerator 1 also by 2 we get

$$\frac{2\times 1}{4}=\frac{2}{4}.$$

But $\frac{2}{4}$ is the same as $\frac{1}{2}$ or the thing we started with. Therefore if we multiply both the numerator and the denominator of $\frac{1}{2}$ by 2, we *do not alter the value of the fraction.*

Shall we alter the value of the fraction $\frac{1}{2}$ if we multiply both numerator and denominator by 3? by 4?

40. What shall we get if we multiply both numerator and denominator of the fraction $\frac{1}{3}$ by 4?

Suppose our $\frac{1}{3}$ to be $\frac{1}{3}$ of a foot (12 inches).

$\frac{1}{3}$ of a foot = 4 inches.

If we multiply the denominator of $\frac{1}{3}$ by 4 we get

$$\frac{1}{4\times 3} \text{ or } \frac{1}{12}.$$

$\frac{1}{12}$ of a foot = 1 inch.

If we now multiply the numerator by 4 we get $\frac{4}{12}$.

$\frac{4}{12}$ of a foot = 4 inches.

But 4 inches is what we started with; therefore we have not altered the value of $\frac{1}{3}$ by multiplying *both* its numerator and its denominator by 4.

Shall we alter the value of the fraction $\frac{1}{3}$ if we multiply both numerator and denominator by 2?

41. If we multiply both numerator and denominator of $\frac{3}{7}$ by 5 we shall get $\frac{15}{35}$; let us compare this result with $\frac{3}{7}$:

$\frac{1}{7} = \frac{5}{35}$; therefore $\frac{3}{7} = 3$ times $\frac{5}{35}$ or $\frac{15}{35}$, and we see that we have not altered the value of the fraction $\frac{3}{7}$ by multiplying both numerator and denominator by 5.

Show that if you multiply both numerator and denominator of $\frac{5}{8}$ by 6 you will not alter the value of the fraction; of $\frac{7}{11}$ by 4; of $\frac{13}{24}$ by 2; of $\frac{5}{19}$ by 6.

In the same way you can take *any* fraction and show that its value will not be changed if you multiply both numerator and denominator by the same number, whatever that number may be.

42. The numerator and denominator of a fraction are sometimes called its *terms*. Thus the terms of the fraction $\frac{3}{7}$ are 3 and 7. By what must you multiply both terms of the fraction $\frac{1}{4}$ in order to get an equivalent fraction whose denominator is 12?

43. By what must you multiply both terms of the fraction $\frac{1}{4}$ in order to change it to eighths? to sixteenths? to twenty-fourths?

44. Multiply both terms of each of the following fractions by something that will change the fraction to sixty-fourths: $\frac{1}{2}$; $\frac{1}{4}$; $\frac{3}{8}$; $\frac{5}{16}$.

45. Change to forty-eighths $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{7}{8}$, $1\frac{1}{2}$, $1\frac{5}{8}$, $1\frac{7}{24}$.

46. What shall we get if we divide both numerator and denominator of $\frac{9}{12}$ by 3?

Suppose our $\frac{9}{12}$ to be $\frac{9}{12}$ of a foot (12 inches).

$\frac{9}{12}$ of a foot = 9 inches.

If we divide the numerator of $\frac{9}{12}$ by 3 we get $\frac{3}{12}$.

$\frac{3}{12}$ of a foot = 3 inches.

If we now divide the denominator of $\frac{3}{12}$ by 3 we get $\frac{3}{4}$.

$\frac{3}{4}$ of a foot = 9 inches.

But 9 inches is what we started with; therefore we have not altered the value of $\frac{8}{12}$ by dividing *both* its numerator and its denominator by 3.

What will you get if you divide both numerator and denominator of $\frac{8}{12}$ by 4?

47. Divide both numerator and denominator of $\frac{12}{15}$ by 3 and compare the result with $\frac{12}{15}$.

$$\frac{12 \div 3}{15 \div 3} = \frac{4}{5}.$$

$\frac{1}{5} = \frac{3}{15}$; therefore $\frac{4}{5} = 4$ times $\frac{3}{15}$ or $\frac{12}{15}$: we have, then, not altered the value of $\frac{12}{15}$ by dividing both numerator and denominator by 3.

Show that if you divide both numerator and denominator of $\frac{18}{20}$ by 4 you will not alter the value of the fraction; of $\frac{20}{40}$ by 10; of $\frac{18}{27}$ by 9; of $\frac{24}{36}$ by 12; of $\frac{21}{28}$ by 7.

In the same way you can take *any* fraction and show that its value will not be changed by dividing both numerator and denominator by the same number.

48. When, as in the last question, we divide both the numerator and denominator by the same number, we are said to reduce the fraction to lower terms; and when there is no number that will go in both the numerator and denominator without a remainder, the fraction is said to have been reduced to its *lowest terms.*

If, for example, we divide both terms of $\frac{8}{12}$ by 2, we reduce it to lower terms and get $\frac{4}{6}$; if we now divide both terms of $\frac{4}{6}$ by 2 we get $\frac{2}{3}$, and have reduced our fraction $\frac{8}{12}$ to its lowest terms, for there is no number (except 1) which will go in both 2 and 3 without a remainder.

Reduce $\frac{15}{20}$ to its lowest terms. *Answer:* $\frac{3}{4}$.

49. Reduce to lowest terms the fractions $\frac{6}{8}$, $\frac{5}{10}$, $\frac{6}{9}$, $\frac{4}{12}$, $\frac{10}{14}$, $\frac{5}{15}$, $\frac{5}{20}$, $\frac{12}{24}$, $\frac{14}{35}$, $\frac{8}{40}$, $\frac{25}{30}$, $\frac{21}{35}$, $\frac{15}{25}$, $\frac{20}{28}$, $\frac{12}{30}$, $\frac{14}{21}$, $\frac{16}{24}$, $\frac{27}{36}$, $\frac{40}{50}$, $\frac{42}{56}$, and $\frac{27}{72}$.

50. Where the denominators of two fractions are the same, the fractions are said to have a *common denominator.* Thus, $\frac{4}{8}$ and $\frac{3}{8}$ have the common denominator 8.

When two fractions have a common denominator we

may add them together by adding the numerators and writing the sum over the common denominator, thus:

$$\frac{4}{9}+\frac{3}{9}=\frac{4+3}{9}=\frac{7}{9}.$$

When we have to add together two fractions which have different denominators we may first change the fractions into equivalent ones which shall have a common denominator, and then add them, thus:

$$\frac{2}{3}+\frac{3}{4}=\frac{8}{12}+\frac{9}{12}=1\frac{7}{12}.$$

$\frac{1}{4}$ is how many twelfths? $\frac{1}{6}$ is how many twelfths? $\frac{1}{4}$ and $\frac{1}{6}$ are how many twelfths?

51. $\frac{1}{4}$ is how many twentieths? $\frac{1}{5}$ is how many twentieths? $\frac{1}{4}$ and $\frac{1}{5}$ are how many twentieths?

52. $\frac{2}{3}$ are how many fifteenths? $\frac{2}{5}$ are how many fifteenths? $\frac{2}{3}$ and $\frac{2}{5}$ are how many fifteenths?

53. $\frac{2}{3}$ are how many sixths? $\frac{2}{3}$ less $\frac{1}{6}$ are how many sixths?

54. Change $\frac{1}{2}$ and $\frac{2}{9}$ to eighteenths and then add them.

Answer: $\frac{1}{2}=\frac{9}{18}$; $\frac{2}{9}=\frac{4}{18}$. Therefore $\frac{1}{2}+\frac{2}{9}=1\frac{3}{18}$.

55. Change $\frac{1}{5}$ and $\frac{1}{7}$ to thirty-fifths and then add them. Which is the larger, and how much?

56. Change $\frac{4}{5}$ and $\frac{2}{3}$ to forty-fifths and then add them. Which is the larger, and how much?

57. A man had a horse, a cow, and a sheep. The horse eat $\frac{2}{3}$ of a load of hay in the winter, the cow $\frac{1}{2}$, and the sheep $\frac{1}{6}$. How many sixths of a load did each eat? how many sixths did they all eat?

58. A boy, having a quart of nuts, wished to divide them so as to give one companion $\frac{1}{2}$, another $\frac{1}{4}$, and a third $\frac{1}{8}$ of them; but, in order to make a proper division, he first separated the whole into eight equal parts and then he was able to divide them as he wished. How many eighths did he give to each? how many eighths had he left for himself?

If we wish to replace two fractions by equivalent fractions having a common denominator we may choose for our common denominator any number which contains both denominators without a remainder.

For instance, if our fractions are $\frac{2}{4}$ and $\frac{3}{9}$ we may choose 36 for our common denominator; for 36 contains both 4 and 9 without a remainder.

If our fractions are $\frac{5}{6}$ and $\frac{1}{4}$ we may choose 12, or 24, or 36, or 48; for each of these contains both 6 and 4 without a remainder, but it is convenient to take as small a number as we can, so in this case we should choose 12.

We can always find a number which will contain both of our denominators without a remainder, by multiplying them together.

59. Reduce $\frac{1}{5}$ and $\frac{3}{7}$ to equivalent fractions having a common denominator.

Solution: $7 \times 9 = 63$ contains both 7 and 9 without a remainder, and may be taken as our common denominator. $\frac{1}{5} = \frac{7}{63}$; $\frac{3}{7} = \frac{27}{63}$.

60. Reduce to a common denominator $\frac{1}{3}$ and $\frac{3}{7}$; $\frac{3}{7}$ and $\frac{2}{5}$; $\frac{1}{2}$ and $\frac{2}{3}$; $\frac{1}{2}$ and $\frac{3}{4}$; $\frac{3}{4}$ and $\frac{5}{6}$.

61. Add together $\frac{2}{3}$ and $\frac{2}{5}$; $\frac{3}{7}$ and $\frac{5}{14}$; $\frac{3}{5}$ and $\frac{5}{15}$; $\frac{1}{2}$ and $\frac{3}{3}$ and $\frac{3}{10}$.

62. Subtract $\frac{1}{3}$ from $\frac{1}{2}$; $\frac{3}{10}$ from $\frac{2}{5}$; $\frac{2}{7}$ from $\frac{2}{3}$; $\frac{2}{5}$ from $\frac{5}{6}$.

63. A boy, distributing some nuts among his companions, gave $\frac{1}{3}$ of a quart to one, and $\frac{1}{2}$ of a quart to another; how much more did he give to one than to the other?

64. What is the difference between $\frac{1}{3}$ and $\frac{1}{2}$?

65. A man, having two bushels of grain to distribute among his laborers, wished to give $\frac{1}{2}$ of a bushel to one, and $\frac{2}{3}$ of a bushel to another, and the rest to a third, but was at a loss to tell how to divide it; at last he concluded to divide each bushel into six equal parts, or sixths, and then to distribute those parts. How many sixths did he give to each?

66. $\frac{2}{3}$ is how many sixths?

67. A man, who had a bushel of wheat, wished to give $\frac{1}{2}$ of it to one man, and $\frac{1}{3}$ of it to another; but he could not tell how to divide it. Another man standing by advised him to divide the whole bushel into six equal parts

first, and then take $\frac{1}{2}$ of them for one, and $\frac{1}{3}$ of them for the other. How many parts did he give to each? how many to both? how many had he left?

68. $\frac{1}{2}$ is how many sixths? $\frac{1}{3}$ is how many sixths? $\frac{1}{2}$ and $\frac{1}{3}$ are how many sixths?

69. $\frac{1}{2}$ and $\frac{1}{4}$ are how many times $\frac{1}{8}$?

70. $\frac{1}{2}$ and $\frac{1}{4}$ are how many times $\frac{1}{4}$?

71. $\frac{2}{3}$ and $\frac{1}{6}$ are how many times $\frac{1}{6}$?

72. $\frac{1}{4}$ and $\frac{5}{8}$ are how many times $\frac{1}{8}$?

73. $\frac{1}{3}$ and $\frac{2}{9}$ are how many times $\frac{1}{9}$?

74. $\frac{1}{2}$ and $\frac{2}{5}$ are how many times $\frac{1}{10}$?

75. $\frac{1}{2}$ and $\frac{1}{4}$ and $\frac{1}{8}$ are how many times $\frac{1}{8}$?

76. $\frac{1}{2}$ and $\frac{3}{5}$ and $\frac{1}{10}$ are how many times $\frac{1}{10}$?

77. $\frac{2}{3}$ and $\frac{1}{4}$ are how many times $\frac{1}{12}$?

78. $\frac{2}{3}$ and $\frac{1}{4}$ and $\frac{1}{6}$ are how many times $\frac{1}{12}$?

79. $\frac{3}{5}$ and $\frac{3}{10}$ and $\frac{1}{4}$ are how many times $\frac{1}{20}$?

80. $\frac{1}{2}$ and $\frac{1}{3}$ and $\frac{3}{4}$ and $\frac{1}{6}$ and $\frac{1}{12}$ are how many times $\frac{1}{24}$?

81. $\frac{1}{2}$ and $\frac{3}{7}$ are how many times $\frac{1}{14}$?

82. $\frac{2}{3}$ and $\frac{1}{5}$ are how many times $\frac{1}{15}$?

83. $\frac{2}{3}$ and $\frac{3}{5}$ are how many times $\frac{1}{15}$?

84. $\frac{2}{3}$ less $\frac{1}{6}$ are how many times $\frac{1}{6}$?

85. Which is the larger, $\frac{3}{4}$ or $\frac{5}{8}$? how much the larger?

86. A boy, having a pound of almonds, said he intended to give $\frac{1}{3}$ of them to his sister, and $\frac{1}{4}$ to his brother, and the rest to his mamma. His mamma, smiling, said she did not think he could divide them so. "Oh, yes, I can," said he; "I will first divide them into twelve equal parts, and then I can divide them well enough." Pray, how many twelfths did he give to each?

87. $\frac{1}{2}$ is how many times $\frac{1}{12}$? $\frac{1}{4}$ is how many times $\frac{1}{12}$? $\frac{1}{3}$ and $\frac{1}{4}$ are how many times $\frac{1}{12}$?

88. Mr. Goodman, having a pound of raisins, said, "I will give Sarah $\frac{1}{3}$, and Mary $\frac{1}{4}$, and James $\frac{1}{6}$ of them, and the rest shall go to Charles, if he can tell how to divide them." "Well," said Charles, "I would first divide the whole into twelve equal parts, and then I could take $\frac{1}{3}$ and $\frac{1}{4}$ and $\frac{1}{6}$ of them." How many twelfths would each have?

89. $\frac{1}{2}$ and $\frac{1}{4}$ and $\frac{1}{8}$ are how many times $\frac{1}{12}$?

90. George bought a pine-apple, and said he would give $\frac{1}{4}$ of it to his papa, and $\frac{2}{5}$ to his mamma, and $\frac{3}{10}$ to his brother James, if James could divide it. James took it, and cut it into twenty equal pieces, and then distributed them as George had desired. How many twentieths did he give to each?

91. $\frac{2}{3}$ and $\frac{3}{4}$ less $\frac{5}{12}$ are how many times $\frac{1}{12}$?

92. $\frac{5}{6}$ less $\frac{2}{3}$ are how many times $\frac{1}{18}$?

93. $\frac{5}{6}$ less $\frac{3}{8}$ are how many times $\frac{1}{24}$?

94. $\frac{4}{5}$ less $\frac{2}{7}$ are how many times $\frac{1}{35}$?

95. $\frac{1}{2}$ and $\frac{3}{4}$ and $\frac{1}{8}$ and $\frac{5}{12}$, less $\frac{5}{8}$, are how many times $\frac{1}{24}$?

96. $\frac{1}{4}$ and $\frac{1}{2}$ and $\frac{2}{5}$ and $\frac{1}{10}$ and $\frac{1}{20}$, less $\frac{7}{8}$, are how many times $\frac{1}{40}$?

97. $\frac{1}{7}$ and $\frac{3}{8}$ are how many times $\frac{1}{42}$?

98. $\frac{3}{8}$ and $\frac{4}{7}$ are how many times $\frac{1}{56}$?

99. $\frac{4}{7}$ and $\frac{2}{9}$ are how many times $\frac{1}{63}$?

100. Mr. Fuller said he would give $\frac{1}{2}$ of a pine-apple to Fanny, and $\frac{2}{5}$ to George, and the rest to the one that could tell how to divide it, and how much there would be left. But neither of them could tell; so he kept it himself. Could you have told, if you had been there? How would you have divided it? how much would have been your share?

101. A man sold $1\frac{1}{2}$ bushels of wheat to one man, and $4\frac{3}{4}$ to another; how many did he sell to both?

102. A man bought $6\frac{1}{3}$ bushels of wheat at one time, and $2\frac{1}{3}$ at another; how much did he buy in all?

103. A man bought $7\frac{2}{3}$ yards of one kind of cloth, and $6\frac{3}{4}$ of another kind; how many yards in all?

104. A man bought $\frac{3}{5}$ of a barrel of flour at one time, and $2\frac{1}{2}$ barrels at another, and $6\frac{3}{4}$ at another; how much did he buy in all?

105. A man bought one sheep for $4\frac{3}{5}$ dollars, and another for $5\frac{3}{7}$ dollars; how much did he give for both?

106. There is a pole standing so that $\frac{1}{3}$ of it is in the mud, and $\frac{2}{5}$ of it in the water, and the rest out of the water; how much of it is out of the water?

107. A man, having undertaken to do a piece of work, did $\frac{1}{2}$ of it the first day, $\frac{1}{3}$ of it the second day, and $\frac{1}{6}$ of it the third day; how much of it did he do in three days?

108. A man having a piece of work to do, hired two men and a boy to do it. The first man could do $\frac{1}{3}$ of the work in a day, and the other $\frac{1}{4}$ of it, and the boy $\frac{1}{8}$ of it; how much of it would they all do in a day?

SECTION X.

Fractions: Miscellaneous Problems and Questions.

A.

1. James had 4 apples, and John had half as many; how many had John?

2. If an orange costs 6 cents, and an apple half as much, how much does the apple cost?

3. If you divide 8 apples equally between two boys, what part of them must each have?

4. What is 1 half of 8?

5. If you divide 8 apples equally among 4 boys, what part of them must each have?

6. What is 1 fourth of 8?

7. If you divide 6 oranges equally among 3 boys, what part of them must 1 boy have?

8. What is 1 third of 6?

9. If 4 yards of cloth cost 8 dollars, what part of 8 dollars would 1 yard cost? What part of 8 dollars would 2 yards cost? What part of 8 dollars would 3 yards cost?

10. What is 1 fourth of 8? What is 2 fourths of 8? What is 3 fourths of 8?

11. If 6 yards of cloth will make 3 coats, what part of 6 yards will make 1 coat? What part of 6 yards will make 2 coats?

12. What is one third of 6? What is two thirds of 6?

13. If 3 barrels of Saratoga water cost 9 dollars, what part of 9 dollars will 1 barrel cost? What part of 9 dollars will 2 barrels cost?

14. What is 1 third of 9? What is 2 thirds of 9?

15. If 2 yards of cloth cost 10 dollars, what part of 10 dollars will 1 yard cost? What part of 10 dollars will 3 yards cost?

16. What is 1 half of 10? What is 3 halves of 10?

17. If 2 barrels of flour cost twelve dollars, what part of twelve dollars will 1 barrel cost? What part of twelve dollars will 3 barrels cost? What part of twelve dollars will 5 barrels cost?

18. What is 1 half of twelve? What is 3 halves of 12? What is 5 halves of twelve?

19. If 2 barrels of cider vinegar cost 12 dollars, what part of 12 dollars will 1 barrel cost? What part of 12 dollars will 3 barrels cost? What part of 12 dollars will 5 barrels cost?

20. What is 1 fourth of 12? What is 2 fourths of 12? 3 fourths? 5 fourths? 7 fourths?

21. If 3 oranges cost 12 cents, what part of 12 cents will 1 orange cost? What part of 12 cents will 2 oranges cost? What part of 12 cents will 4 oranges cost? What part of 12 cents will 5 oranges cost? 7 oranges? 10 oranges?

22. What is 1 third of 12? 2 thirds? 4 thirds? 5 thirds? 7 thirds? 10 thirds?

23. If 8 bushels of wheat cost 10 dollars, what part of 10 dollars will 1 bushel cost? What part of 10 dollars will 2 bushels cost? 3 bushels? 4 bushels? 6 bushels? 7 bushels?

24. What is 1 fifth of 10? 2 fifths? 3 fifths? 4 fifths? 7 fifths?

25. What is 1 half of 14? 1 seventh? 2 sevenths? 3 sevenths? 5 sevenths?

26. What is 1 third of 15? 2 thirds? 1 fifth? 2 fifths? 3 fifths? 4 fifths?

27. What is 1 half of 16? 3 fourths? 1 eighth? 3 eighths? 5 eighths? 7 eighths? 1 sixteenth?

28. What is 1 half of 18? 2 thirds? 1 sixth? 5 sixths? 1 ninth? 2 ninths? 4 ninths? 5 ninths? 7 ninths? 8 ninths? 1 eighteenth? 5 eighteenths?

29. What is 1 half of 20? 1 fourth? 3 fourths? 1 fifth? 3 fifths? 2 fifths? 4 fifths? 1 twentieth? 3 twentieths? 7 twentieths?

30. What is 1 third of 21? 2 thirds? 1 seventh? 5 sevenths? 3 sevenths? 6 sevenths? 4 sevenths?

31. What is 1 half of 22? 1 eleventh? 3 elevenths? 5 elevenths? 7 elevenths? 10 elevenths?

32. What is 1 half of 24? 1 third? 2 thirds? 1 fourth? 3 fourths? 1 sixth? 5 sixths? 5 eighths? 3 eighths? 7 eighths? 1 twelfth? 5 twelfths? 7 twelfths?

33. What is 1 half of 26?

34. What are 2 thirds of 27? 2 ninths? 4 ninths? 5 ninths? 8 ninths? 7 ninths?

35. What is 1 half of 28? 3 fourths? 2 sevenths?

36. What is 1 sixth of 30? 3 fifths? 7 tenths? 2 thirds? 1 half?

37. What is 1 half of 32? 3 fourths? 5 eighths? 1 sixteenth?

38. What is 1 half of 34?

39. What is 5 sixths of 36? 2 thirds? 1 half? 7 twelfths? 4 ninths? 3 fourths?

40. What is 1 half of 38?

41. What is 1 third of 39?

42. What is 3 fourths of 40? 7 eighths? 9 tenths? 1 half?

43. What is 3 sevenths of 42? 5 sixths?

B.

1. A man bought 6 oranges at 4 cents apiece; how many cents did they come to? He paid for them with cherries at 8 cents a pint; how many pints did it take?

2. How many times 8 are 6 times 4?

3. A man bought 3 yards of cloth at 4 dollars a yard; how many dollars did it come to? How much flour at 6 dollars a barrel would it take to pay for it?

4. How many times 6 are 3 times 4?

5. A man bought 4 peaches at 3 cents apiece; how many cents did they come to? He paid for them with pears at 2 cents apiece; how many pears did it take?

6. How many times 2 are 4 times 3?

7. Mr. Brown bought 2 hundred-weight of sugar at 12 dollars a hundred-weight, and paid for it with wood at 6 dollars a cord; how many cords did it take?

8. How many times 6 are 2 times 12?

9. I bought 3 barrels of flour at 8 dollars a barrel, and paid for it with cider vinegar at 6 dollars a barrel; how many barrels did it take?

10. How many times 6 are 3 times 8?

11. How many times 5 are 12 times 3?

Answer: 12 times 3 are 36; 5 in 36 goes $7\frac{1}{5}$ times. Therefore 12 times 3 are $7\frac{1}{5}$ times 5.

12. How many times 8 are 6 times 4?

13. How many times 6 are 3 times 10?

14. How many times 6 are 4 times 9?

15. How many pints of berries at 4 cents a pint must James give for 3 packages of torpedoes at 5 cents a package?

16. How many times 4 are 3 times 5?

17. How many times 3 are 2 times 7? how many times 5? 4?

18. 4 times 5 are how many times 3? 6? 7?

19. 3 times 7 are how many times 4? 5? 6? 8? 9?

20. I bought 2 kegs and 2 sevenths of a keg of gunpowder at 7 dollars a keg, and paid for it with wood at 8 dollars a cord; how many cords did it take? How much butter at 4 dollars a box would it have taken to pay for it?

21. 2 times 7 + 2 sevenths of 7 are how many times 4? 3? 5? 6? 8?

22. I bought $3\frac{3}{4}$ quarts of salt at 5 cents a quart, and paid for it with milk at 6 cents a quart; how much milk did it take?

23. 3 times 5 + 3 fifths of five are how many times 6? 9? 4? 7? 3? 8?

24. How much sugar that is 11 dollars a hundredweight, can be bought for 4 cords and 2 sevenths of a cord of wood at 7 dollars a cord?

25. 4 times 7 + 2 sevenths of 7 are how many times 6? 8? 5? 3? 9? 10?

26. 5 times 5 + 3 fifths of 5 are how many times 4? 8? 9? 7? 10? 3? 6?

27. 6 times 7 + 3 sevenths of 7 are how many times 9? 4? 5? 8? 10?

28. 5 times 8+3 eighths of 8 are how many times 6? 9? 4? 7? 10?

29. 7 times 8+5 eighths of 8 are how many times 9? 6? 10? 4? 5?

30. 5 times 9+4 ninths of 9 are how many times 7? 8? 6? 10? 4?

31. 7 times 9+7 ninths of 9 are how many times 6? 8? 10? 5? 4?

32. 6 times 10+3 tenths of 10 are how many times 7? 5? 4? 9? 8?

33. 8 times 10+4 tenths of 10 are how many times 6? 7? 9?

34. 8 times 9+3 ninths of 9 are how many times 6? 10? 7?

35. A boy, having 12 apples, kept 1 fourth of them himself, and divided the other 3 fourths equally among 4 of his companions; how many did he give to each?

36. 3 fourths of 12 are how many times 4?

37. A man having 14 bushels of grain, divided 5 sevenths of it equally among 3 men; how much did he give to each?

38. 5 sevenths of 14 are how many times 3?

39. A man who had 15 feet of rope gave away 4 fifths of it; how many fathoms did he give away, if 6 feet make a fathom?

40. 4 fifths of 15 are how many times 6?

41. A man having 21 cents, paid away 6 sevenths of them for oranges at 5 cents apiece; how many oranges did he buy?

42. 6 sevenths of 21 are how many times 5?

43. A country grocer said to a farmer, "I will give you 3 gallons of kerosene for 5 pounds of maple sugar." On these terms the farmer took 24 gallons of kerosene. How much sugar did the grocer receive, and how much did he get for it if he sold it at the rate of 6 pounds for a dollar?

44. 5 thirds of 24 are how many times 6?

45. 7 fifths of 20 are how many times 6?

46. 5 sevenths of 35 are how many times 8?

47. 7 sixths of 36 are how many times 5?
48. 9 sevenths of 42 are how many times 8?
49. 7 ninths of 45 are how many times 6?
50. 5 sixths of 48 are how many times 7?
51. 6 fifths of 50 are how many times 9?
52. 8 ninths of 54 are how many times 5?
53. 9 sevenths of 56 are how many times 10?
54. 7 sixths of 60 are how many times 8?
55. 9 sevenths of 63 are how many times 7?
56. 10 eighths of 64 are how many times 9?
57. 6 eighths of 72 are how many times 7?
58. 4 sevenths of 84 are how many times 9?
59. 7 tenths of 90 are how many times 8?

C.

1. Charles had 6 apples, and gave 1 third of them to John; how many did he give him?
2. Albert had 9 cents, and spent 2 thirds of them; how many had he left?
3. James had 10 pears, and gave 1 half of them to one of his companions, and 2 fifths of them to another; how many did he give away?
4. If 3 yards of cloth cost 6 dollars, what will one yard cost?
5. If 4 yards of cloth cost 12 dollars, what will 2 yards cost?
6. If 9 apples cost 18 cents, what will 3 apples cost?
7. If 3 oranges cost 18 cents, what will 2 cost?
8. James had 25 cents, and gave 4 fifths of them for 10 apples; how much did he give for all the apples? how much apiece?
9. A man had 30 dollars, and gave 5 sixths of them for 8 yards of cloth; how much did he give a yard?
10. A man had 40 yards of cloth, and sold 3 fifths of it for 24 dollars; what was that a yard?
11. A man had 42 barrels of flour, and sold 2 sevenths of it for 8 dollars a barrel; how much did he receive?

12. A boy had 63 nuts, and divided 4 sevenths of them equally among six of his companions; how many did he give them apiece?

13. If 4 yards of cloth cost 12 dollars, what will 3 yards cost?

14. If 5 oranges cost 25 cents, what will 3 cost?

15. If 3 oranges cost 15 cents, what will 7 cost?

16. If 3 barrels of cider vinegar cost 18 dollars, what will 10 barrels cost?

17. If 7 pounds of flour cost 35 cents, what will 9 pounds cost?

18. If 5 firkins of butter cost 40 dollars, what will 3 firkins cost?

19. If 2 men do a piece of work in 6 days, how long would it take 4 men to do the same work?

20. If 6 men can do a piece of work in 12 days, in how many days will 3 men do the same work?

21. If 3 men can do a piece of work in 12 days, in how many days will 4 men do the same work?

22. If 2 pipes of a certain size will empty a cistern in 6 hours, in how long a time will 3 pipes of the same size empty it?

23. Three men, setting out on a journey, purchased 5 loaves of bread apiece, but, before they had eaten any of it, two other men joined them: it was then agreed to share the bread equally among the whole; how many loaves did each man receive?

24. If 3 barrels of flour cost twenty-four dollars, what would 7 barrels cost? How much cider vinegar at 7 dollars a barrel would 6 barrels of flour buy?

25. A man bought a quantity of flour for fifty-four dollars, and another man gave him 9 yards of cloth for five sixths of it; what was the cloth worth a yard?

26. If 9 yards of cloth cost fifty-four dollars, how many boxes of butter, at 4 dollars a box, would 5 yards of the same cloth buy?

27. I bought 8 firkins of butter for seventy-two dollars, and gave 6 of them for 7 yards of cloth; what was a yard of the cloth worth?

28. A man bought 6 barrels of flour for sixty dollars, and gave 4 barrels of it for vinegar at 5 dollars a barrel; how many barrels of vinegar did he get?

29. If $\frac{1}{8}$ of a cask of whale oil costs $15, what would a cask cost? How much vinegar, at $6 a barrel, would it take to pay for a cask of the oil?

30. 15 is 5 eighths of how many times 6?

[First find of what number 15 is 5 eighths; and then find how many times this number contains 6. Thus: If 15 is five eighths, 1 eighth will be 1 fifth of 15 or 3, and 8 eighths will be 8 times 3 or 24; 24 is 3 times 6. Therefore 15 is 5 eighths of 3 times 6.]

31. If 2 thirds of a pound of coffee cost 24 cents, how much would a pound cost? How many pints of peanuts, at 6 cents a pint, might be bought for a pound?

32. 24 is 2 thirds of how many times 6?

33. A man bought $\frac{1}{3}$ of a pound of coffee for 20 cents; how much would a pound come to at this rate?

34. 20 is 4 sevenths of how many times 6?

[20 is 4 sevenths of 35; and 35 is 5 and 5 sixths times 6.]

35. A man sold a cow for 21 dollars, which was only 7 tenths of what she cost him; how much did she cost him? When he bought her, he paid for her with cloth at 4 dollars a yard; how many yards of cloth did he give?

36. 21 is 7 tenths of how many times 4?

37. A man, being asked the age of his youngest son, answered that the age of his eldest son was 24 years, which was 3 fifths of his own age, and that his own age was 10 times as much as that of his youngest son; what was his age? and what was the age of his youngest son?

38. 24 is 3 fifths of how many times 10?

39. 27 is 3 fifths of how many times 7?

40. 28 is 7 tenths of how many times 9?

41. 30 is 5 eighths of how many times 7?

42. 32 is 4 sevenths of how many times 6?

43. 36 is 9 eighths of how many times 5?

44. 40 is 8 ninths of how many times 8?

45. 42 is 6 fifths of how many times 4?
46. 45 is 9 eighths of how many times 6?
47. 48 is 8 ninths of how many times 7?
48. 50 is 5 sevenths of how many times 8?
49. 54 is 9 sixths of how many times 7?
50. 56 is 7 ninths of how many times 10?
51. 60 is 10 sevenths of how many times 4?
52. 63 is 9 eighths of how many times 5?
53. 64 is 8 ninths of how many times 7?
54. 70 is 10 sevenths of how many times 8?
55. 72 is 9 fifths of how many times 6?
56. 80 is 10 thirds of how many times 4?
57. 80 is 8 fifths of how many times 6?

58. A boy gave away 4 cents, which was 1 third of all he had; how many had he at first?

59. A boy gave 5 apples to one of his companions, which was 1 fourth of what he had; how many had he?

60. A man paid away 4 dollars, which was 2 thirds of all the money he had; how much had he?

61. A man sold a watch for 18 dollars, which was 3 fourths of what it cost him; how much did it cost?

62. A man sold a calf for 15 dollars, which was 3 fifths of what the calf cost; how much did he lose by his bargain?

63. A man bought 12 yards of cloth, and sold it for 54 dollars, which was 9 eighths of what it cost him; what did it cost him a yard? and how much did he gain by his bargain?

64. There is a pole standing in the water, so that 10 feet of it is above the water, which is 2 thirds of the whole length of the pole; how long is the pole?

65. There is a pole 2 thirds under water, and 4 feet out; how long is the pole?

Answer: Since there are 3 thirds in all, and since 2 thirds are under water, the part that is out must be 1 third; if 1 third is 4 feet, 3 thirds, or a whole one, will be 3 times 4 feet, or 12 feet. Therefore the pole is 12 feet long.

66. There is a pole 2 fifths under water, and 6 feet out of the water; how long is the pole?

67. There is an orchard, in which 3 sevenths of the trees bear cherries, and 2 sevenths bear peaches, and 10 trees bear plums; how many trees are there in the orchard? and how many of each sort?

68. There is a school, in which, from 9 o'clock to 10 in the morning, 2 ninths of the boys learn arithmetic, 3 ninths learn grammar, 1 ninth learn geography, 1 ninth learn geometry, and 12 learn to write; how many are there in the school?

69. A man sold a watch for 63 dollars, which was 7 fifths of what it cost him; how much did he gain?

D.

1. If a yard of cloth costs 3 dollars, what will 1 half of a yard cost?

2. What is 1 half of 3?

3. If a barrel of vinegar costs 8 dollars, what will 1 half of a barrel cost?

4. If 2 barrels of Congress water cost 7 dollars, what will a barrel cost?

5. What is $\frac{1}{2}$ of 7? of 9? 11? 13? 15?

6. If you divide 1 bushel of wheat equally among 3 persons, what part of a bushel will you give to each?

7. If 3 yards of cloth cost 1 dollar, what part of a dollar will 1 yard cost?

8. What is 1 third of 1?

9. How can you divide 2 oranges into 3 equal parts, that is, how can you find 1 third of 2 oranges?

10. One third of 2 oranges will be the same as how many thirds of 1 orange?

11. If you divide 2 bushels of wheat equally among 3 persons, what part of a bushel will you give to each?

12. If 3 bushels of corn cost 2 dollars, what part of a dollar will 1 bushel cost?

13. If 3 bushels of wheat cost 4 dollars, how much will a bushel cost?

14. What is one third of 2? of 4?

15. What is 1 third of 5? of 7? of 8? of 10? of 11?

16. If a bushel of apples be divided equally among 4 persons, what part of a bushel will they have apiece? What would they have apiece, if 2 bushels were divided among them? what if 3 bushels? what if 5 bushels? what if 6 bushels?

17. What is 1 fourth of 1? of 2? of 3? of 5? of 6? of 7? of 9? of 10?

18. If a bushel of corn-meal will serve 5 persons 1 month, how much will serve 1 person the same time?

19. If 3 barrels of flour be divided among 5 men, how much will each have? If 4 barrels were divided, what would each have? what if 6 barrels were divided? what if 7 barrels were divided?

20. What is 1 fifth of 1? of 2? of 3? of 4? of 6? of 7?

21. What is 1 sixth of 1? of 2? of 3? of 4? of 5? of 7? of 8? of 9? of 10?

22. What is 1 seventh of 1? of 2? of 3? of 4? of 5? of 6?

23. What is 1 eighth of 1? of 2? of 3? of 4? of 5? of 6? of 7? of 8? of 9? of 10?

24. What is 1 ninth of 1? of 2? of 3? of 4? of 5? of 6? of 7? of 8? of 9? of 10? of 11?

25. What is 1 tenth of 1? of 2? of 3? of 4? of 5? of 6? of 7? of 8? of 9? of 10? of 11? of 12? of 13?

26. If 3 yards of cloth cost 2 dollars, what will 1 yard cost? What will 2 yards cost?

27. If 1 bushel of seed-wheat costs 2 dollars, what will 1 third of a bushel cost? What will 2 thirds of a bushel cost?

What is $\frac{1}{3}$ of 2? $\frac{2}{3}$ of 2?

28. If a load of wood costs 5 dollars, what will 1 third of a load cost? What will 2 thirds of a load cost?

29. What is $\frac{1}{3}$ of 5? $\frac{2}{3}$ of 5? $\frac{2}{3}$ of 7?

30. If 4 bushels of salt cost 3 dollars, what will 1 bushel cost? What will 3 bushels cost?

31. What is $\frac{1}{4}$ of 3? $\frac{3}{4}$ of 3?

32. What is $\frac{1}{2}$ of 2? $\frac{2}{3}$ of 2? $\frac{1}{3}$ of 7? $\frac{3}{4}$ of 7?

33. What is $\frac{1}{4}$ of 5? $\frac{3}{4}$ of 5? $\frac{1}{8}$ of 4? $\frac{5}{8}$ of 4?

34. If you divide 6 dollars among 8 men, what part of a dollar will you give to each? What would three of them have?

35. What is $\frac{1}{8}$ of 6? $\frac{3}{8}$ of 7? $\frac{1}{9}$ of 10? $\frac{7}{9}$ of 10? $\frac{1}{10}$ of 14? $\frac{4}{10}$ of 14?

36. If 5 yards of cloth cost 17 dollars, what is that a yard? What would 3 yards cost? What would 8 yards cost?

37. What is $\frac{2}{5}$ of 17? $\frac{3}{5}$ of 17? $\frac{4}{7}$ of 20? $\frac{3}{5}$ of 22? $\frac{3}{8}$ of 27? $\frac{3}{4}$ of 28? $\frac{3}{4}$ of 31?

38. If 5 loads of hay cost 47 dollars, what is that a load? What will 2 loads cost? What will 7 loads cost? What will 12 loads cost?

39. What is $\frac{2}{5}$ of 47? $\frac{4}{5}$ of 47? $\frac{12}{15}$ of 47? $\frac{5}{7}$ of 48? $\frac{4}{5}$ of 50? $\frac{3}{7}$ of 58? $\frac{3}{8}$ of 61? $\frac{4}{10}$ of 73? $\frac{5}{8}$ of 65?

40. If you divide 7 apples equally between 2 boys, how many would you give them apiece?

41. How can you divide 5 oranges equally among 3 persons?

42. If you were to divide 3 bushels of corn equally among 5 men, how much would you give to each?

43. A boy had 7 pears, and gave away 3 fifths of them; how many did he give away? and how many had he left?

44. If 2 yards of cloth cost 3 dollars, what is that a yard?

45. If 4 yards of cloth cost 2 dollars, what is that a yard?

46. If 5 bushels of corn cost 7 dollars, what is that a bushel?

47. If a man receives 8 dollars for 6 days' work, what does he receive per day?

48. If 3 bushels of grass-seed cost 8 dollars, what will 2 bushels cost?

49. A man had 30 dollars, and gave away 3 sevenths of it; how much did he give away?

50. If 4 yards of cloth cost 10 dollars, what will 3 yards cost?

51. If 7 pounds of flour cost 40 cents, what will 10 pounds cost?

52. If 4 firkins of butter cost 26 dollars, what will 7 firkins cost?

53. If 3 men can do a piece of work in 7 days, how long will it take 1 man to do it? How long would it take 4 men?

54. If 2 pipes will empty a cistern in 3 hours, in how long a time would 1 pipe empty it? In how long a time would 7 pipes empty it?

E.

1. In 2 how many times $\frac{1}{2}$?
2. In 3 how many times $\frac{1}{2}$?
3. In 2 how many times $\frac{1}{3}$?
4. In 4 how many times $\frac{1}{3}$?
5. In 6 how many times $\frac{1}{4}$?
6. In 7 how many times $\frac{1}{8}$?
7. In 8 how many times $\frac{1}{5}$?
8. In $2\frac{1}{2}$ how many times $\frac{1}{2}$?
9. In $3\frac{1}{4}$ how many times $\frac{1}{4}$?
10. Reduce $4\frac{1}{5}$ to an improper fraction.
11. Reduce $3\frac{2}{7}$ to an improper fraction.
12. Reduce $5\frac{3}{8}$ to an improper fraction.
13. Reduce $6\frac{2}{3}$ to an improper fraction.
14. Reduce $8\frac{3}{10}$ to an improper fraction.
15. Reduce $9\frac{4}{7}$ to an improper fraction.

16. $\frac{4}{2}$ are how many times 1?
17. $\frac{6}{3}$ are how many times 1?
18. $\frac{7}{7}$ are how many times 1?
19. $\frac{8}{4}$ are how many times 1?
20. $\frac{9}{9}$ are how many times 1?
21. $\frac{6}{1}$ are how many times 1?
22. $\frac{8}{1}$ are how many times 1?
23. $\frac{3}{1}$ are how many times 1?

24. $\frac{2}{3}$ are how many times 1?
25. $\frac{3}{10}$ are how many times 1?

F.

1. How much is 3 times $\frac{1}{5}$?
2. How much is 4 times $\frac{1}{3}$?
3. How much is 3 times $\frac{2}{5}$?
4. How much is 4 times $\frac{2}{7}$?
5. How much is 5 times $\frac{3}{8}$?
6. How much is 6 times $\frac{4}{5}$?
7. How much is 8 times $\frac{3}{10}$?
8. How much is 9 times $\frac{5}{7}$?
9. How much is 10 times $\frac{5}{8}$?
10. How much is 9 times $\frac{5}{8}$?

11. How much is 3 times $2\frac{1}{5}$?
12. How much is 4 times $3\frac{2}{3}$?
13. How much is 5 times $6\frac{4}{7}$?
14. How much is 6 times $4\frac{5}{8}$?
15. How much is 7 times $5\frac{2}{7}$?
16. How much is 8 times $6\frac{4}{5}$?
17. How much is 4 times $10\frac{3}{7}$?
18. How much is 9 times $7\frac{4}{5}$?
19. How much is 8 times $9\frac{4}{5}$?
20. How much is 10 times $7\frac{3}{5}$?

G.

1. What is $\frac{1}{5}$ of 6?
2. What is $\frac{1}{3}$ of 6?
3. What is $\frac{1}{4}$ of 8?
4. What is $\frac{1}{9}$ of 9?
5. What is $\frac{2}{3}$ of 9?
6. What is $\frac{1}{5}$ of 10?
7. What is $\frac{2}{7}$ of 14?
8. What is $\frac{1}{3}$ of 5?
9. What is $\frac{3}{5}$ of 5?
10. What is $\frac{1}{5}$ of 7?
11. What is $\frac{2}{5}$ of 7?
12. What is $\frac{3}{7}$ of 35?
13. What is $\frac{1}{5}$ of 17?
14. What is $\frac{2}{7}$ of 26?
15. What is $\frac{5}{8}$ of 27?
16. What is $\frac{4}{5}$ of 37?
17. What is $\frac{6}{7}$ of 47?
18. What is $\frac{3}{4}$ of 42?
19. What is $\frac{4}{7}$ of 65?
20. What is $\frac{6}{7}$ of 75?

H.

1. 2 is $\frac{1}{2}$ of what number?
2. 4 is $\frac{1}{3}$ of what number?
3. 8 is $\frac{1}{5}$ of what number?
4. $1\frac{1}{2}$ is $\frac{1}{3}$ of what number?
5. $2\frac{3}{5}$ is $\frac{1}{3}$ of what number?
6. $4\frac{2}{7}$ is $\frac{1}{4}$ of what number?
7. $6\frac{4}{5}$ is $\frac{1}{7}$ of what number?
8. $7\frac{6}{7}$ is $\frac{1}{8}$ of what number?
9. $8\frac{4}{5}$ is $\frac{1}{8}$ of what number?
10. $9\frac{3}{10}$ is $\frac{1}{10}$ of what number?

11. 4 is $\frac{2}{3}$ of what number?
12. 6 is $\frac{3}{4}$ of what number?
13. 8 is $\frac{4}{5}$ of what number?
14. 12 is $\frac{3}{7}$ of what number?
15. 15 is $\frac{5}{8}$ of what number?
16. 18 is $\frac{6}{7}$ of what number?
17. 20 is $\frac{5}{9}$ of what number?
18. 24 is $\frac{8}{9}$ of what number?
19. 28 is $\frac{7}{9}$ of what number?
20. 30 is $\frac{6}{10}$ of what number?

21. 3 is $\frac{2}{3}$ of what number?
22. 4 is $\frac{3}{5}$ of what number?
23. 5 is $\frac{4}{7}$ of what number?
24. 8 is $\frac{3}{8}$ of what number?
25. 9 is $\frac{4}{7}$ of what number?
26. 17 is $\frac{2}{5}$ of what number?
27. 25 is $\frac{4}{9}$ of what number?
28. 38 is $\frac{3}{5}$ of what number?
29. 43 is $\frac{2}{7}$ of what number?
30. 54 is $\frac{10}{9}$ of what number?

SECTION XI.

The Multiplication and Division of Fractions.

A. Multiplication of a Fraction by a Fraction.

1. A boy having $\frac{1}{2}$ of an apple gave away $\frac{1}{2}$ of what he had; what part of the whole apple did he give away?

Answer: $\frac{1}{2}$ of an apple is the same as $\frac{2}{4}$ of an apple [See § 8, A., page 104]; therefore the boy gave away $\frac{1}{2}$ of $\frac{2}{4}$ of an apple or $\frac{1}{4}$ of an apple.

2. What is $\frac{1}{2}$ of $\frac{1}{2}$?

3. If you cut an orange into three equal pieces, and then cut each of those pieces into two equal pieces, how many pieces will the whole orange be cut into? What part of the whole orange will one of the pieces be?

4. What is $\frac{1}{2}$ of $\frac{1}{3}$?

5. A boy had $\frac{1}{2}$ of a pine-apple, and cut that half into three equal pieces, in order to give away $\frac{1}{3}$ of it. What part of the whole apple did he give away?

6. What is $\frac{1}{3}$ of $\frac{1}{2}$?

7. If an orange is cut into 4 parts, and then each of the parts is cut in two, how many pieces will the whole be cut into?

8. What is $\frac{1}{2}$ of $\frac{1}{4}$?

9. A man having $\frac{1}{2}$ of a barrel of flour, sold $\frac{1}{4}$ of what he had; how much did he sell?

10. What is $\frac{1}{4}$ of $\frac{1}{2}$?

11. If an orange be cut into 4 equal parts, and each of these parts be cut into 3 equal parts, into how many parts will the whole orange be cut?

12. What is $\frac{1}{3}$ of $\frac{1}{4}$?

13. A boy having $\frac{1}{3}$ of a quart of chestnuts, gave away $\frac{1}{4}$ of what he had. What part of the whole quart did he give away?

14. What is $\frac{1}{4}$ of $\frac{1}{3}$?

15. What is $\frac{1}{2}$ of $\frac{1}{5}$?

16. A man who owned $\frac{1}{3}$ of a ship, sold $\frac{1}{3}$ of his

share; what part of the ship did he sell, and what part did he then own?

[When we get $\frac{1}{3}$ of $\frac{1}{5}$ we are said *to multiply the fractions together;* we can, then, use the multiplication sign ($\times$) instead of the word "of," thus $\frac{1}{3} \times \frac{1}{5}$.]

17. What is $\frac{1}{3} \times \frac{1}{5}$?
18. What is $\frac{1}{4} \times \frac{1}{5}$?
19. What is $\frac{1}{2} \times \frac{1}{6}$?
20. What is $\frac{1}{5}$ of $\frac{1}{8}$?
21. What is $\frac{1}{4}$ of $\frac{1}{7}$?
22. What is $\frac{1}{3}$ of $\frac{1}{8}$?
23. What is $\frac{1}{6}$ of $\frac{1}{5}$?
24. What is $\frac{1}{4}$ of $\frac{1}{6}$?
25. A boy having $\frac{2}{3}$ of an orange (that is, 2 pieces), gave his sister $\frac{1}{2}$ of what he had; how many thirds did he give her?
26. What is $\frac{1}{2}$ of $\frac{2}{3}$?
27. A boy, having $\frac{3}{4}$ of a pine-apple, said he would give one half of what he had to his sister, if she could tell how to divide it. His sister said, "You have $\frac{3}{4}$, or three pieces, if you cut them all in two, you can give me $\frac{1}{2}$ of each one of them." How much did his sister receive?
28. What is $\frac{1}{2}$ of $\frac{3}{4}$?
29. A man who owned $\frac{3}{4}$ of a share in a Boston bank, sold $\frac{1}{3}$ of his part; what part of a share did he sell?
30. What is $\frac{1}{3}$ of $\frac{3}{4}$?
31. A man who owned $\frac{4}{5}$ of a ship, sold $\frac{1}{4}$ of his share; what part of the whole ship did he sell? What part had he left?
32. What is $\frac{1}{4}$ of $\frac{4}{5}$?
33. What is $\frac{1}{3}$ of $\frac{6}{7}$?
34. What is $\frac{1}{4}$ of $\frac{12}{7}$?
35. What is $\frac{1}{2}$ of $\frac{3}{7}$?

Answer: $\frac{1}{2}$ of $\frac{1}{7} = \frac{1}{14}$, therefore $\frac{1}{2}$ of $\frac{3}{7}$ will be 3 times as much, or $\frac{3}{14}$.

36. A man who owned $\frac{5}{8}$ of a share in a bank, sold $\frac{1}{3}$ of his part; what part of a whole share did he sell?
37. What is $\frac{1}{3}$ of $\frac{5}{8}$?

38. What is $\frac{1}{4}$ of $\frac{3}{7}$?

39. A boy having $\frac{4}{5}$ of a watermelon, wished to divide his part equally among his sister, his brother, and himself, but was at a loss to know how to do it; but his sister advised him to cut each of the fifths into 3 equal parts. How many pieces did each have? and what part of the whole melon was each piece?

40. What is $\frac{1}{3}$ of $\frac{4}{5}$?

41. What is $\frac{1}{5}$ of $\frac{4}{7}$?

42. What is $\frac{1}{3}$ of $\frac{1}{4}$?

43. What is $\frac{2}{3}$ of $\frac{1}{4}$?

44. What is $\frac{1}{4}$ of $\frac{2}{3}$?

45. What is $\frac{2}{4}$ of $\frac{2}{5}$?

Answer: We first get $\frac{1}{4}$ of $\frac{2}{5}$. Now $\frac{1}{4}$ of $\frac{1}{5} = \frac{1}{20}$; $\frac{1}{4}$ of $\frac{2}{5}$ is, then, twice as much, or $\frac{2}{20}$, which equals $\frac{1}{10}$. Therefore $\frac{3}{4}$ of $\frac{2}{5}$, which is 3 times $\frac{1}{4}$ of $\frac{2}{5}$, is $\frac{3}{10}$.

46. What is $\frac{1}{3}$ of $\frac{2}{5}$?

47. What is $\frac{2}{3}$ of $\frac{2}{5}$?

48. What is $\frac{1}{3}$ of $\frac{5}{6}$?

49. What is $\frac{2}{3}$ of $\frac{5}{6}$?

50. What is $\frac{5}{6}$ of $\frac{3}{4}$?

51. What is $\frac{1}{5}$ of $\frac{3}{7}$?

52. What is $\frac{4}{5}$ of $\frac{3}{7}$?

53. What is $\frac{1}{10}$ of $\frac{3}{8}$?

54. What is $\frac{7}{10}$ of $\frac{3}{8}$?

55. If a yard of cloth costs $2\frac{1}{2}$ dollars, what will $\frac{1}{2}$ of a yard cost?

Answer: If 1 yard costs $\$2\frac{1}{2}$ or $\$\frac{5}{2}$, $\frac{1}{2}$ a yard will cost $\frac{1}{2}$ as much or $\$\frac{5}{4}$, which is $\$1\frac{1}{4}$.

56. What is $\frac{1}{2}$ of $2\frac{1}{2}$?

57. A boy had $2\frac{1}{2}$ oranges, and wished to give $\frac{1}{3}$ of them to his sister, and $\frac{1}{3}$ to his brother, but he did not know how to divide them equally. His brother told him to cut the whole oranges into halves, and then cut each of the halves into 3 pieces. What part of a whole orange did each have?

58. What is $\frac{1}{3}$ of $2\frac{1}{2}$?

59. A man bought 4 bushels of corn for $3\frac{2}{3}$ dollars; what part of a dollar did 1 bushel cost?

Answer: $\$3\frac{2}{3} = \$1\frac{1}{3}$. If 4 bushels cost $\$1\frac{1}{3}$, one bushel, $\frac{1}{4}$ of 4 bushels, will cost $\frac{1}{4}$ of $\$1\frac{1}{3}$ or $\$\frac{1}{3}$.

60. What is $\frac{1}{4}$ of $5\frac{2}{3}$?

61. If 5 bushels of wheat cost $7\frac{3}{4}$ dollars, what is that a bushel?

62. What is $\frac{1}{5}$ of $7\frac{3}{4}$?

63. A man bought 6 gallons of alcohol for $8\frac{2}{3}$ dollars; what was that a gallon?

64. What is $\frac{1}{6}$ of $8\frac{2}{5}$?

65. A man bought 7 bushels of potatoes for $8\frac{4}{7}$ dollars; how much was that a bushel?

66. What is $\frac{1}{7}$ of $8\frac{4}{7}$?

67. A man bought 10 pieces of nankeen for $6\frac{3}{8}$ dollars; how much was it a piece?

68. What is $\frac{1}{10}$ of $6\frac{3}{8}$?

69. If 9 bushels of rye cost $6\frac{3}{5}$ dollars, what is that a bushel?

70. What is $\frac{1}{5} \times 7\frac{2}{3}$?

71. What is $\frac{1}{8} \times 5\frac{3}{7}$?

72. What is $\frac{1}{3} \times 8\frac{4}{5}$?

73. What is $\frac{1}{4} \times 6\frac{3}{10}$?

74. What is $\frac{1}{5} \times 9\frac{4}{7}$?

75. A man bought 7 yards of cloth for $18\frac{3}{5}$ dollars; what was that a yard? What would 3 yards cost?

76. What is $\frac{1}{7}$ of $18\frac{3}{5}$? What is $\frac{3}{7}$ of $18\frac{3}{5}$?

77. A man bought 5 barrels of vinegar for $27\frac{3}{8}$ dollars; what was it a barrel? What would 7 barrels cost at that rate?

78. What is $\frac{1}{5}$ of $27\frac{3}{8}$? What is $\frac{4}{5}$ of $27\frac{3}{8}$?

79. If 6 barrels of flour cost $38\frac{4}{5}$ dollars, what would 10 barrels cost at that rate?

80. What is $\frac{10}{8} \times 38\frac{4}{5}$?

81. Show that $\frac{1}{3} \times \frac{1}{5} = \frac{1}{5 \times 3} = \frac{1}{15}$.

If we divide $\frac{1}{3}$ into 3 equal parts, one of the parts will be $\frac{1}{3}$ of $\frac{1}{3}$ $(\frac{1}{3} \times \frac{1}{3})$. If in $\frac{1}{3}$ there are 3 of these parts, in 5 fifths there will be 5×3, or 15 parts; one part, then, is $\frac{1}{5 \times 3}$ or $\frac{1}{15}$. Therefore $\frac{1}{3} \times \frac{1}{5} = \frac{1}{5 \times 3} = \frac{1}{15}$.

82. Show that $\frac{1}{3}\times\frac{4}{5}=\frac{4}{5\times3}=\frac{4}{15}$.

$\frac{1}{3}\times\frac{4}{5}$ will be 4 times as much as $\frac{1}{3}\times\frac{1}{5}$.

Now $\frac{1}{3}\times\frac{1}{5}=\frac{1}{5\times3}=\frac{1}{15}$. (See last Question.)

Therefore $\frac{1}{3}\times\frac{4}{5}=\frac{4}{5\times3}=\frac{4}{15}$.

83. Show that

$\frac{1}{3}\times\frac{1}{4}=\frac{1}{3\times4}=\frac{1}{12}$;
$\frac{1}{3}\times\frac{1}{2}=\frac{1}{3\times2}=\frac{1}{6}$;
$\frac{1}{3}\times\frac{2}{3}=\frac{2}{3\times3}=\frac{2}{9}$;
$\frac{1}{3}\times\frac{2}{5}=\frac{2}{3\times5}=\frac{2}{15}$.

To get $\frac{1}{3}$ of a fraction, that is, to multiply it by $\frac{1}{3}$, we may multiply its denominator by 3.

84. Show that $\frac{2}{3}\times\frac{4}{5}=\frac{2\times4}{5\times3}=\frac{8}{15}$.

We know from Question 82 that $\frac{1}{3}\times\frac{4}{5}=\frac{4}{5\times3}=\frac{4}{15}$.

Now $\frac{2}{3}\times\frac{4}{5}$ is 2 times as much as $\frac{1}{3}\times\frac{4}{5}$;

therefore $\frac{2}{3}\times\frac{4}{5}=\frac{2\times4}{5\times3}=\frac{8}{15}$.

85. Show that

$\frac{2}{3}\times\frac{2}{5}=\frac{2\times2}{3\times5}=\frac{4}{15}$;
$\frac{2}{3}\times\frac{2}{3}=\frac{2\times2}{3\times3}=\frac{4}{9}$;
$\frac{2}{3}\times\frac{5}{7}=\frac{2\times5}{3\times7}=\frac{10}{21}$.

To get $\frac{2}{3}$ of a fraction, that is, to multiply it by $\frac{2}{3}$, we multiply the numerator by 2, and the denominator by 3.

86. Show that

$\frac{2}{5}\times\frac{6}{7}=\frac{2\times6}{5\times7}=\frac{12}{35}$;
$\frac{2}{9}\times\frac{2}{3}=\frac{2\times2}{9\times3}=\frac{4}{27}$;
$\frac{3}{2}\times\frac{3}{4}=\frac{3\times3}{2\times4}=\frac{9}{8}$;
$\frac{4}{3}\times\frac{2}{5}=\frac{4\times2}{3\times5}=\frac{8}{15}$.

To multiply two fractions together, we multiply the numerators together to get a new numerator, and the denominators together to get a new denominator.

87. What is $\frac{1}{6} \times \frac{1}{4}$? $\frac{1}{3} \times \frac{1}{7}$? $\frac{1}{5} \times \frac{1}{7}$? $\frac{1}{7} \times \frac{1}{9}$?
88. What is $\frac{1}{2} \times \frac{2}{5}$? $\frac{1}{3} \times \frac{2}{7}$? $\frac{1}{5} \times \frac{3}{4}$? $\frac{1}{8} \times \frac{4}{5}$? $\frac{1}{10} \times \frac{4}{9}$?
89. What is $\frac{2}{7} \times \frac{4}{5}$? $\frac{3}{8} \times \frac{2}{3}$? $\frac{5}{8} \times \frac{3}{10}$? $\frac{7}{5} \times \frac{2}{9}$? $\frac{6}{9} \times \frac{8}{7}$?
90. What is $1\frac{1}{2} \times 2\frac{2}{3}$? $1\frac{1}{2} \times 3\frac{1}{4}$? $3\frac{1}{3} \times 4\frac{1}{2}$? $5\frac{1}{5} \times \frac{5}{13}$? $\frac{1}{6} \times 2\frac{1}{7}$?

91. A man bought a piece of cloth for $42\frac{3}{7}$ dollars, and was obliged to sell it for $\frac{4}{5}$ of what it cost him; how much did he lose?

92. A man bought a quantity of flour for $53\frac{3}{7}$ dollars, and sold it for $\frac{7}{8}$ of what it cost him; how much did he gain?

93. If 7 men can do a piece of work in $4\frac{2}{3}$ days, how long will it take 1 man to do it? How long will it take 3 men to do it?

94. If 4 men can do a piece of work in $9\frac{3}{7}$ days, how long would it take to do it if 7 men were employed?

95. There is a pole standing so that $\frac{3}{7}$ of it is in the water, and $\frac{2}{5}$ as much in the mud; how much is in the mud?

96. If a man can travel $13\frac{4}{5}$ miles in 3 hours, how many miles can he travel in 8 hours?

97. If 5 horses can eat $26\frac{7}{8}$ loads of hay in a year, what will 8 horses eat in the same time?

98. If 4 pipes can empty a cistern in $6\frac{4}{5}$ hours, how long will it take 7 pipes of the same size to empty it?

B. To Divide by a Fraction.

1. A boy having 2 oranges, wished to give $\frac{1}{3}$ of an orange apiece to his playmates; how many could he give them to? If he had given $\frac{2}{3}$ of an orange apiece, how many could he have given them to?

Answer: If he gives $\frac{1}{3}$ of an orange to each playmate, he can give to as many playmates as there are *thirds* in 2, or to 6 playmates.

Had he given $\frac{2}{3}$ of an orange to each, he could have given to only half as many, or to 3 playmates.

Or, we may find the answer to the last question in this

way: In two oranges there are 6 thirds; if he gives 2 thirds to each, he can give to as many playmates as there are twos in 6, or to 3 playmates.

2. How many times $\frac{1}{3}$ are there in 2? How many times $\frac{2}{3}$ are there in 2?

3. A man having 3 bushels of corn, distributed it among some poor persons, giving $\frac{3}{4}$ of a bushel to each; to how many did he give it?

Answer: In 3 bushels there are 12 *fourths* bushels. Had he given 1 fourth to each, he could have given to 12 persons; if, then, he gives 3 fourths to each, he can give to only $\frac{1}{3}$ as many, or to 4 persons.

Or, since in 3 bushels there are 12 fourths bushels, he can give 3 fourths to as many persons as 3 is contained in 12, or to 4 persons.

4. In 3 are how many times $\frac{1}{4}$? how many times $\frac{3}{4}$?

5. If $\frac{2}{5}$ of a barrel of flour will last a family one month, how long will 4 barrels last the same family? How long will 6 barrels last? How long will 10 barrels last?

6. How many times is $\frac{1}{2}$ contained in 4? how many times in 6? how many times in 10?

7. If $\frac{3}{4}$ of a bushel of wheat will last a family one week, how many weeks will $6\frac{3}{4}$ bushels last the same family?

Answer: In $6\frac{3}{4}$ bushels there are $\frac{27}{4}$ bushels; at the rate of $\frac{1}{4}$ in a week, the whole would last 27 weeks; and at the rate of $\frac{3}{4}$ in a week, the whole would last $\frac{1}{3}$ of 27 weeks, or 9 weeks.

Or, $6\frac{3}{4}$ bushels $= \frac{27}{4}$ bushels; at the rate of $\frac{3}{4}$ in a week, $\frac{27}{4}$ will last as many weeks as 3 is contained in 27, or 9 weeks.

8. How many times is $\frac{2}{3}$ contained in $6\frac{2}{3}$?

9. There is a cistern having a pipe which will fill it in $\frac{2}{3}$ of an hour; how many times would the pipe fill the cistern in $3\frac{2}{3}$ hours? *Answer:* $1\frac{1}{2}$, or $8\frac{1}{2}$ times.

10. How many times is $\frac{2}{5}$ contained in $3\frac{2}{5}$?

11. How much cloth at $1\frac{1}{2}$ dollars (that is, $\frac{3}{2}$ dollars) a yard can be bought for 4 dollars?

12. How many times is $1\frac{1}{2}$ or $\frac{3}{2}$ contained in 4?

13. How many barrels of potatoes at $\$1\frac{1}{2}$ a barrel can I buy for $\$8\frac{1}{2}$?

14. How many times is $1\frac{1}{2}$ contained in $8\frac{1}{2}$?

15. If a soldier is allowed $1\frac{1}{3}$ pounds (that is $\frac{4}{3}$ of a pound) of meat in a day, to how many soldiers would $6\frac{2}{3}$ pounds be allowed?

16. How many times is $1\frac{1}{3}$ contained in $6\frac{2}{3}$?

17. If $1\frac{2}{3}$ tons of hay will keep a horse through the winter, how many horses will 10 tons keep?

18. How many times is $1\frac{2}{3}$ contained in 10?

19. At $2\frac{1}{3}$ dollars a box, how many boxes of raisins can be bought for 10 dollars?

20. How many times is $2\frac{1}{3}$ contained in 10?

21. At $1\frac{3}{8}$ dollars a pound, how many pounds of indigo can be bought for $9\frac{3}{8}$ dollars?

22. How many times is $1\frac{3}{8}$ contained in $9\frac{3}{8}$?

23. At $1\frac{3}{4}$ dollars a barrel, how many barrels of raisins can be bought for $9\frac{3}{4}$ dollars?

24. How many times is $1\frac{3}{4}$ contained in $9\frac{3}{4}$?

25. At $\frac{7}{8}$ of a dollar apiece, how many pieces of nankeen can be bought for $8\frac{3}{8}$ dollars?

26. How many times is $\frac{7}{8}$ contained in $8\frac{3}{8}$?

27. At $\frac{5}{8}$ of a dollar a pound, how many pounds of tea can be bought for $7\frac{3}{8}$ dollars?

28. How many times is $\frac{5}{8}$ contained in $7\frac{3}{8}$?

29. How many times is $3\frac{1}{5}$ contained in $7\frac{3}{5}$?

30. How many times is $5\frac{1}{2}$ contained in 17?

31. How many times is $4\frac{1}{5}$ contained in $9\frac{3}{5}$?

32. How many times is $3\frac{3}{7}$ contained in $12\frac{4}{7}$?

33. At $\frac{1}{10}$ of a dollar a pound, how many pounds of meat can be bought for $\frac{1}{2}$ of a dollar?

Note. Change $\frac{1}{2}$ to tenths.

34. How many times is $\frac{1}{10}$ contained in $\frac{1}{2}$?

35. If a man can do $\frac{1}{8}$ of a piece of work in one hour, how many hours will it take him to do $\frac{3}{4}$ of the work?

Note. Change both fractions to twelfths; that is, reduce them to a common denominator.

36. How many times is $\frac{1}{8}$ contained in $\frac{3}{4}$?

37. If a pound of almonds cost $\frac{1}{7}$ of a dollar, how many pounds can be bought for $\frac{3}{5}$ of a dollar?

Note. Reduce the fractions to a common denominator.

38. How many times is $\frac{1}{7}$ contained in $\frac{3}{5}$?

39. If a piece of nankeen costs $\frac{3}{5}$ of a dollar, how many pieces can be bought for $4\frac{3}{4}$ dollars, that is, for $\frac{19}{4}$ dollars?

40. How many times is $\frac{3}{5}$ contained in $4\frac{3}{4}$?

41. If a bushel of barley costs $\frac{2}{5}$ of a dollar, how many bushels can be bought for $\frac{1}{3}$ of a dollar? How many for $1\frac{5}{8}$ dollars?

42. How many times is $\frac{3}{5}$ contained in $\frac{1}{3}$? How many times in $1\frac{5}{8}$?

43. How many times is $\frac{3}{7}$ contained in $\frac{3}{5}$?

44. How many times is $\frac{2}{3}$ contained in $\frac{3}{8}$?

45. Show that $\frac{1}{3}$ is contained

in 1, 3 times;
in 2, 6 times;
in 3, 9 times;
in 4, 12 times;
in 5, 15 times.

} To divide a whole number by $\frac{1}{3}$ we multiply the number by 3.

46. Show that $\frac{1}{5}$ is contained

in 1, 5 times;
in 2, 10 times;
in 3, 15 times;
in 4, 20 times.

} To divide a whole number by $\frac{1}{5}$ we multiply the number by 5.

47. Show that $\frac{2}{5}$ is contained

in 1, $\frac{5}{2}$ times;*
in 2, $\frac{10}{2}$ or 5 times;
in 3, $\frac{15}{2}$ times;
in 4, $\frac{20}{2}$ or 10 times.

} To divide a whole number by $\frac{2}{5}$ we may multiply the number by 5 and divide by 2.

48. Show that

$$2 \div \tfrac{3}{5} = \tfrac{3}{5} \times 2 = \tfrac{6}{5} = 3;$$
$$2 \div \tfrac{3}{4} = \tfrac{4}{3} \times 2 = \tfrac{8}{3};$$
$$2 \div \tfrac{5}{6} = \tfrac{6}{5} \times 2 = \tfrac{12}{5},$$
$$2 \div \tfrac{3}{7} = \tfrac{7}{3} \times 2 = \tfrac{14}{3}.$$

* $\frac{2}{5}$ will be contained only $\frac{1}{2}$ as many times as $\frac{1}{5}$.

49. Show that

$1 \div \frac{3}{5} = \frac{5}{3}$ $\left[\frac{3}{3} \times 1\right]$
$2 \div \frac{3}{5} = \frac{10}{3}$ $\left[\frac{3}{3} \times 2\right]$
$3 \div \frac{1}{4} = 12$ $\left[\frac{1}{1} \times 3\right]$
$5 \div \frac{4}{7} = \frac{35}{4}$ $\left[\frac{7}{4} \times 5\right]$

To divide a whole number by a fraction we may multiply by the denominator and divide by the numerator, or, as is sometimes said, *we may invert the divisor and proceed as in multiplication.*

In the last example, for instance, 7×5 or 35 [the number of sevenths in 5] is the number of times that $\frac{1}{7}$ is contained in 5, and $\frac{1}{7}$ will be contained $\frac{1}{4}$ as many times or $\frac{35}{4}$ times.

50. What is $7 \div \frac{3}{5}$? $4 \div \frac{3}{5}$? $6 \div \frac{2}{5}$? $20 \div \frac{4}{5}$? $21 \div \frac{7}{5}$? $18 \div \frac{3}{11}$? $16 \div \frac{8}{5}$?

51. We have already shown that

$$2 \div \frac{2}{3} = \frac{3}{2} \times 2 = \frac{6}{2} = 3;$$
$$2 \div \frac{3}{4} = \frac{4}{3} \times 2 = \frac{8}{3};$$
$$2 \div \frac{5}{6} = \frac{6}{5} \times 2 = \frac{12}{5};$$
$$2 \div \frac{3}{7} = \frac{7}{3} \times 2 = \frac{14}{3}.$$

If now we divide $\frac{1}{5}$ of 2 or $\frac{2}{5}$ by the same divisors the results will be only $\frac{1}{5}$ as large.

Show, then, that

$\frac{2}{5} \div \frac{2}{3} = \frac{3}{2} \times \frac{2}{5} = \frac{6}{10} = \frac{3}{5}$;
$\frac{2}{5} \div \frac{3}{4} = \frac{4}{3} \times \frac{2}{5} = \frac{8}{15}$;
$\frac{2}{5} \div \frac{5}{6} = \frac{6}{5} \times \frac{2}{5} = \frac{12}{25}$;
$\frac{2}{5} \div \frac{3}{7} = \frac{7}{3} \times \frac{2}{5} = \frac{14}{15}$.

TO DIVIDE ONE FRACTION BY ANOTHER WE MAY INVERT THE DIVISOR AND PROCEED AS IN MULTIPLICATION.

SECTION XII.

Tables.

A. English Money.

12 pence = 1 shilling.
20 shillings = 1 pound sterling.

1. How many pence are there in 2 shillings? in 4 shillings? in 1 shilling and 8 pence? in 5 shillings and 6 pence? in 10 shillings? in 1 pound? in 1 pound, 2 shillings, and 6 pence?

2. How many shillings are there in 2 pounds? in 5 pounds? in 3 pounds and 8 shillings? in 4 pounds and 10 shillings? in 10 pounds?

3. A sixpenny piece is what part of a shilling? of a pound?

4. A two-shilling piece is what part of a pound? of 2 pounds and 10 shillings?

5. How many 5 shilling pieces could you get for 1 pound? for 3 pounds? for 7 pounds and 10 shillings?

6. Instead of writing out the words "pounds sterling" after a number, it is usual to express them by a £ written before the number, thus: £12 stands for 12 pounds sterling; £32 stands for 32 pounds sterling.

The word "shillings" is generally abbreviated to *s.* and the word "pence" is expressed by *d.* (the first letter of the Latin word *denarius*, meaning a penny).

Thus: £8 2*s.* 6*d.* stands for 8 pounds, 2 shillings, and 6 pence.

Read £4 3*s.* 2*d.*; £15 8*s.* 3*d.*; £196 10*s.*; £2143 0*s.* 6*d.*

7. Write in the same way,

One hundred and fourteen pounds;

Eight pounds, four shillings, and six pence;

Fourteen hundred pounds, ten shillings, and two pence;

Twelve pounds, three shillings, and seven pence.

8. How many pence are equivalent to £1 2*s.* 6*d.*?

Answer: 270*d.* This is called "reducing a sum of money to pence."

9. Reduce £4 3*s.* 4*d.* to pence.

10. Reduce £8 10*s.* to shillings; to pence.

11. Reduce £14 8*s.* to shillings.

12. How many pounds could you get for 80 shillings? for 140 shillings?

13. How many shillings could you get for 84 pence? for 72 pence?

14. Show that 87*d.* are equivalent to 7*s.* 3*d.* and that 281*d.* are equivalent to £1 3*s.* 5*d.*

This is called reducing pence to pounds, shillings, and pence.

15. Show that 194*s.* are equivalent to £9 14*s.*

This is called reducing shillings to pounds and shillings.

16. Reduce 64 shillings to pounds and shillings.

Solution: There are 20 shillings in a pound; there are, then, in 64 shillings as many pounds as 20 is contained in 64, or 3 pounds with a remainder of 4 shillings. Therefore 64*s.* = £3 4*s.*

17. Reduce 86*s.* to pounds and shillings; 98*s.*; 123*s.*; 249*s.*

18. Reduce 391*d.* to pounds, shillings, and pence.

Solution: There are 12 pence in a shilling: in 391*d.*, then, there are as many shillings as 12 is contained in 391, or 32 shillings with a remainder of 7 pence. Therefore 391*d.* = 32*s.* 7*d.*

Now 32*s.* are equivalent to £1 12*s.*; therefore 391*d.* = 32*s.* 7*d.* = £1 12*s.* 7*d.* *Answer.*

19. Reduce 408*d.* to pounds, shillings, and pence; 256*d.*; 504*d.*; 642*d.*

20. At 10*d.* a gallon, how much will 12 gallons of petroleum cost?

21. At 3½*d.* a quart, how much must I pay for 8 quarts of milk?

22. When flour sells for 32*s.* 10*d.* per barrel, how much can I get for 8 barrels?

B. Liquid Measure.

4 gills = 1 pint.
2 pints = 1 quart.
4 quarts = 1 gallon.

1. What part of a gallon is a quart? a pint? a gill?

2. How many pints are there in 8 gallons and 3 quarts? in 6 gallons, 2 quarts, and 1 pint?

3. How many quarts are there in 25 gallons and 3 quarts?

4. Prove that 23 pints are equivalent to 11 quarts and 1 pint; or to 2 gallons, 3 quarts, and 1 pint.

5. Show that 87 gills = 21 pints and 3 gills
= 10 quarts, 1 pint, and 3 gills
= 2 gallons, 2 quarts, 1 pint, and 3 gills.

6. Show that 402 pints = 201 quarts
= 50 gallons, 1 quart.

7. Mrs. Jones, the confectioner, made $4\frac{1}{2}$ gallons of ice cream one hot day last summer at a cost of \$0.40 per quart, and sold it all in plates at \$0.10 per gill; how much did she make on her ice cream that day?

8. That same day Mrs. Jones sold 8 gallons of lemonade, which had cost her 20 cents a gallon to make, for 5 cents per half pint glass; how much did Mrs. Jones make on her lemonade?

9. Mr. Brown pays each morning 34 cents a can for 24 cans of milk holding $8\frac{1}{2}$ quarts each. He carries this milk to town and sells it to his customers for 7 cents a quart; how much does he get for his trouble?

10. Mr. Southwick paid \$5.76 for a barrel containing 32 gallons of vinegar, and sold the vinegar for 30 cents a gallon; how much did he make in all? How much did he make on a quart? how much on a pint?

11. If a gallon of alcohol costs \$2.88, what is the cost of a gill?

C. Dry Measure.

2 pints = 1 quart.
8 quarts = 1 peck.
4 pecks = 1 bushel.

1. What part of a bushel is a peck? a quart? a pint?

2. How many quarts are there in 4 bushels? in 6 bushels and a peck?

3. How many pints are there in 3 pecks? in 2 bushels? in 3 bushels and 3 quarts?

4. Show that 37 pecks = 9 bushels and 1 peck.

5. Show that 93 quarts = 11 pecks and 5 quarts
= 2 bushels, 3 pecks, 5 quarts.

6. Show that 165 pints = 82 quarts and 1 pint
= 10 pecks, 2 quarts, and 1 pint
= 2 bushels, 2 pecks, 2 quarts, and 1 pint.

7. Show that there are 214 pints in 3 bushels, 1 peck, and 3 quarts.

8. Mr. Whipple bought 8 bushels of new potatoes for $1 a bushel and sold them at retail for 6 cents a quart; how much did he make in all?

9. During the berry season last year, which lasted 3 weeks, James picked 18 quarts of berries a week and sold them at 56 cents a peck; how much did he earn in all?

10. Mr. Briggs, the miller, bought enough corn for $28 to make 20 bushels of meal which he sold at 7 cents a quart; how much did he make by the transaction?

11. Mr. Foster has 108 bushels of corn. If he gives each of his 3 horses 6 quarts a day how long will this corn last?

12. Mr. Breck bought 2 bushels of grass-seed for $3 a bushel, paid 30 cents for paper bags into which to put the seed, and sold it for 15 cents a quart; how much did he make by the operation?

D. Weight.

16 ounces = 1 pound.
2000 pounds = 1 ton.

1. Oz. is used to stand for the word ounce or ounces, and lb. for the word pound. Thus, 6 oz. stands for 6 ounces; and 8 lbs. stands for 8 pounds.

How many ounces are there in 7 pounds? in 21 lbs. 3 oz.? in 123 lbs. 9 oz.?

2. Show that 178 oz. = 11 lbs. 2 oz.

3. If the postage upon printed matter is a cent for every 2 ounces, what must I pay upon a book which weighs 2 lbs. and 4 oz.?

4. Mr. Southwick bought 218 lbs. of beef for 13 cents per lb., and sold it at an average price of 19 cents per lb.; how much did he make by the operation?

5. What will 2 lbs. 5 oz. of quicksilver cost at 48 cents per lb.?

6. Mr. Bartlett, the druggist, bought a pound of sachet powder for $1.60, which he put up in half-ounce packets and sold for 10 cents a packet; how much did he make on the whole?

7. If a basket of coal weighs 80 lbs., how many baskets will there be in the 2-ton load which the coal-dealer has just sent to me?

8. Mr. Hubbard took a load of hay to Boston and sold it at $18 a ton: the hay and the wagon weighed 5900 lbs.; after the hay was unloaded the wagon was found to weigh 1800 lbs. How much did Mr. Hubbard get for his load of hay?

E. Long Measure.

12 inches = 1 foot.
3 feet = 1 yard.
5½ yards or 16½ feet = 1 rod.
320 rods or 5280 feet = 1 mile.

1. What part of a yard is an inch? How many inches are there in a quarter of a yard?

2. What part of $2\frac{1}{2}$ miles is a rod?

3. How many feet are equivalent to half a mile? to a quarter of a mile?

4. How many yards are there in a mile? in a quarter of a mile?

5. How many feet are there in 516 yards? in 12 rods?

6. How many yards are there in 40 rods? in 1 mile and 12 rods?

7. Show that 88 feet = 29 yards and 1 foot.

8. Show that 99 yards = 18 rods.

9. Show that 4132 inches = 344 feet, 4 inches
= 114 yards, 2 feet, and 4 inches.

10. At $3.25 a yard, what will a piece of cloth containing 44 yards cost?

11. Mr. Glines has a square field of such a size that each one of the four sides is 36 rods long; how many feet of fencing will it take to surround the field?

12. Wire musquito-netting of the width of my windows costs 20 cents a yard; how much must I pay for enough to make 6 window nets, each 1 foot 3 inches high?

13. Mr. Knapp wants 4 pieces of iron rod to stay his chimneys with; each piece is to be fifteen feet long, and of such a size that a piece 10 inches long weighs a pound; what must he pay for the whole at 12 cents a pound?

F. Measure of Time.

60 seconds = 1 minute.
60 minutes = 1 hour.
24 hours = 1 day.
7 days = 1 week.
$365\frac{1}{4}$ days = 1 year.
366 days = 1 leap year.

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Excepting February alone,
To which we twenty-eight assign,
Till leap year gives it twenty-nine.

1. What part of an hour is a second?
2. How many minutes are there in a day? how many seconds?
3. How many hours are there in a week?
4. If a certain steam-engine makes one revolution in every two seconds, how many revolutions will it make in an hour?
5. If a steamship goes a mile in 4 minutes, how far will she go in a day? how long at this rate would it take her to make a voyage of 3000 miles?
6. How many minutes in 4 hours and 28 minutes?
7. If a train starts at 16 minutes past 2 o'clock and reaches its destination at 28 minutes past 4 o'clock, how long is it on the way?
8. In the yacht race, the Swallow started at 4 minutes past 1 o'clock and reached the finish at 27 minutes before 6 o'clock; how long was she in going over the course?
9. It is now the 17th of August, and John's school begins on the 3d of September, how many days more of vacation are there?

The last day of school was the 23d of June, how long is the summer vacation?

G. Miscellaneous Questions and Problems.

1. In 2 pounds how many ounces?
2. In 8 yards how many feet?
3. In 3 quarters of a yard how many inches?
4. $\frac{3}{10}$ of a dollar is how many cents?
5. How many pence is $\frac{5}{6}$ of a shilling?
6. In £$\frac{3}{8}$ how many shillings?

7. How many pence is $\frac{5}{8}$ of a shilling?
8. How many pecks are there in $\frac{4}{5}$ of a bushel?
9. How many hours are there in $\frac{5}{8}$ of a day?
10. How many minutes are there in $\frac{3}{5}$ of a day?
11. How many hours are there in $\frac{3}{7}$ of a week?
12. How many seconds are there in $\frac{2}{3}$ of an hour?
13. If a man spends 28 dollars in a month, what does he spend in a week? in a day?
14. If a man spends 21 dollars a week, what does he spend in a day?
15. If a man buys 4 bushels of grain for 5 dollars, how much does he pay for a peck?
16. If alcohol is $2\frac{1}{2}$ dollars a gallon, how much is it a pint?
17. If you give 5 cents a gill for varnish, what should you give for a pint? what for a quart? what for a gallon?
18. If alcohol is worth 30 cents a pint, what will a gill cost? a quart? a gallon?
19. If a yard of cloth is worth 6 dollars, what are $2\frac{3}{8}$ yards worth?
20. If a man earns $10\frac{1}{2}$ dollars a week, what can he earn in a day? what in 3 days? what in $4\frac{1}{2}$ days?
21. If a man earns $2\frac{3}{5}$ dollars in a day, what will he earn in a week?
22. 1 penny is what part of a shilling?
23. 2 pence is what part of a shilling?
24. 3 pence is what part of a shilling?
25. 5 pence is what part of a shilling?
26. 6 pence is what part of a shilling?
27. 7 pence is what part of a shilling?
28. 8 pence is what part of a shilling?
29. 9 pence is what part of a shilling?
30. 10 pence is what part of a shilling?
31. 11 pence is what part of a shilling?
32. 1 shilling is what part of a pound?
33. 2 shillings is what part of a pound?
34. 3 shillings is what part of a pound?
35. 4 shillings is what part of a pound?

36. 5 shillings is what part of a pound?

37. What part of a pound is 6 shillings? 7 shillings? 8 shillings? 9 shillings? 10 shillings? 11 shillings? 12 shillings? 13 shillings? 14 shillings? 15 shillings? 16 shillings? 17 shillings? 18 shillings? 19 shillings?

38. How many pence are there in a pound?

39. One penny is what part of a pound?

40. What part of a pound is 2 pence? 3 pence? 4 pence? 5 pence? 6 pence? 7 pence? 8 pence? 11 pence? 15 pence? 27 pence? 35 pence?

41. How many pence are there in 1 shilling and 6 pence?

42. In 2 shillings and 4 pence, how many pence?

43. In 4 shillings and 5 pence, how many pence?

44. In 5 shillings and 8 pence, how many pence?

45. In 9 shillings and 11 pence, how many pence?

46. What part of £1 is 2*s.* 6*d.*?

[Reduce the whole to pence.]

47. 3*s.* 6*d.* is what part of £1?

48. 7*s.* 8*d.* is what part of £1?

49. What is the price of 2 yards of cloth, at 3*s.* 4*d.* a yard?

50. What will 8 yards of cloth cost, at 2*s.* 8*d.* a yard?

51. What will 4 bushels of wheat cost, at 5*s.* 9*d.* a bushel?

52. What must you give for 4 barrels of vinegar, at $6\frac{1}{4}$ dollars a barrel?

53. If we divide 3 bushels of wheat between 2 men, how many pecks will each receive?

54. If you divide 4 bushels of corn among 5 men, how much will you give to each?

55. If you divide 3 bushels of corn among 7 men, how much will you give to each?

56. How many inches are there in 1 yard?

57. How many inches are there in 4 yards?

58. How many inches are there in 5 yards and 2 inches?

59. In 7 yards and 3 quarters, how many quarters?

60. In $4\frac{1}{2}$ yards, how many inches?

61. 1 inch is what part of a quarter of a yard?
62. 3 inches is what part of a quarter of a yard?
63. 1 inch is what part of a yard?
64. What part of 1 yard is 3 inches? 5 inches? 7 inches? 10 inches? 15 inches?
65. In 8 feet how many yards?
66. In 12 quarters of a yard how many yards?
67. In 10 quarters of a yard how many yards?
68. In 96 inches how many yards?
69. In 12 inches how many quarters of a yard?
70. In 16 inches how many quarters of a yard? how many yards?
71. In 24 inches how many quarters of a yard? how many yards?
72. In 35 quarters of a yard how many yards?
73. In 45 inches how many yards?
74. In 63 inches how many yards?
75. At 2 cents an inch, what would 4 yards of cloth cost?
76. At $2\frac{2}{3}$ dollars for 1 quarter of a yard, what would 2 yards cost?
77. 1 oz. is what part of a lb.?
78. What part of a lb. is 2 oz.? 3 oz.? 4 oz.? 5 oz.? 7 oz.? 10 oz.? 15 oz.?
79. At 3 cents for 1 oz. what would 1 lb. cost?
80. At 3 cents for 2 oz. what would 1 lb. cost?
81. At 3 cents for 8 oz. what would 1 lb. cost?
82. At 5 cents for 10 oz. what would 1 lb. cost?

SECTION XIII.

Miscellaneous Questions and Problems.

A.

1. How many apples, at 2 cents apiece, must you give for 2 lemons at 4 cents apiece?

[The 2 lemons come to 8 cents, and 8 cents will buy 4 apples at 2 cents apiece.]

2. How many pears, at 3 cents apiece, must you give for 3 oranges at 5 cents apiece?

3. How many barrels of apples, at 3 dollars a barrel, must be given for 5 boxes of butter at 4 dollars a box?

4. A man bought 4 yards of broadcloth at 7 dollars a yard, and paid for it with flour at 5 dollars a barrel; how many barrels did he give?

5. If 2 apples cost 4 cents, what would 4 apples cost?

6. If 3 apples are worth 6 cents, how many apples must you give for 8 pears that are worth 3 cents apiece?

7. James had 8 oranges that were worth 5 cents apiece; George had 5 quarts of cherries that were worth 6 cents a quart, which he gave to James for a part of his oranges; how many oranges did George buy, and how many had James left?

8. An Englishman bought 5 barrels and 3 sevenths of a barrel of salt at 7 shillings a barrel; how many pounds did it come to?

9. A grocer bought 9 boxes and 2 thirds of a box of raisins for 3 dollars a box, and paid for it with vinegar at 4 dollars a barrel; how many barrels did it take?

10. Mr. Marr bought 8 pounds and 4 sevenths of a pound of opium at $3\frac{1}{2}$ dollars a pound, and paid for it with cloth at 5 dollars a yard; how many yards did it take?

11. A fruit dealer bought 6 tons and 4 ninths of a ton of Cannel coal at 9 dollars a ton, and paid for it with

oranges at 5 dollars a box; how many boxes did it take?

12. Mr. Jones bought 7 cases and 5 sixths of a case of Florence oil at 6 dollars a case, and paid for it with sheet-lead at 7 dollars a hundred-weight; how many hundred-weight did it take?

13. How many dozen of eggs, at 24 cents a dozen, must you give for 7 pounds of sugar, at 10 cents a pound?

14. How much barley, at 3 shillings a bushel, must be given for 8 bushels of wheat at 7 shillings a bushel?

15. How much cloth, at 4 shillings a yard, must be given for a firkin of butter worth 48 shillings?

16. How much cloth, at 5 shillings a yard, can be bought for 2 reams of paper at 30 shillings a ream?

17. How much wheat, at 7 shillings a bushel, can be bought for 2 barrels of pears at 26 shillings a barrel?

18. How long would it take a man to lay up 60 shillings if he saves 4 shillings a day?

19. If a man earns 8 shillings a day, how many shillings would he earn in 10 days?

20. A man bought twenty pears at the rate of 2 for 3 cents; how much did they come to?

[2 pears are contained in 20 pears 10 times. 10 times 3 cents are 30 cents. Or, first find what 20 pears would come to at 3 cents apiece; and since it is 2 for 3 cents, instead of 1 for 3 cents, the price will be half as much.]

21. How many eggs, at the rate of 3 for 8 cents, can you buy for forty-eight cents?

[See how many times you can have 8 cents in 48 cents, and you can buy so many times 3 eggs.]

22. A man hired a laborer, and agreed to give him 5 dollars for every 3 days' work; how much did he give him a week, there being 6 working-days in a week? how much a month, allowing 4 weeks to the month?

23. If a man receives 5 dollars for 3 days' work, how many quarters of a dollar is that a day?

24. 5 men bought a horse for sixty-three dollars, and paid two dollars a week for keeping him; at the end of

8 weeks they sold him for fifty-four dollars; how much did each man lose by the bargain?

Answer: 5 dollars.

25. If 1 yard of cloth costs 4 dollars, what will 5 yards cost?

26. A man bought 3 pounds of raisins at 12 cents a pound, and 16 oranges at 4 cents apiece, and 1 pound of candles for 16 cents; what did they all come to?

27. A boy had 37 apples; he kept 5 himself, and divided the rest equally among four companions; how many did he give them apiece?

28. Two men are 40 miles apart, and both traveling the same way: the hindermost man gains upon the other 5 miles each day; in how many days will he overtake him?

29. Two men are traveling the same way; one travels at the rate of 38 miles a day; the other, who is behind the former, travels 44 miles a day; how much does he gain on the first each day? and in how many days would he gain 60 miles?

30. A fox is 80 rods before a greyhound, and is running at the rate of 27 rods in a minute; the greyhound is following at the rate of 31 rods in a minute; in how many minutes will the greyhound overtake the fox?

[The greyhound gains upon the fox 4 rods in a minute. How long will it take him to gain 80 rods?]

31. If 2 yards of cloth cost 6 dollars, what would 4 yards cost? what would 12 yards cost?

32. If 8 sheep cost 24 dollars, what would 3 cost?

[$\frac{3}{8}$ of 24. Or, first find the cost of 1 sheep, and then of 3 sheep.]

33. If 4 tons of hay will keep 3 horses through the winter, how many tons will keep 30 horses the same time?

[30 horses will eat how many times as much as 3 horses?]

34. If a man spends a dollar and a quarter a day, how many dollars will he spend in a week?

35. Mr. Taylor bought ten pieces of cloth, each containing 5 yards, for 100 dollars; what was it a piece? and how much a yard?

36. If the wages of 12 weeks come to 60 dollars, what is that a month? and how much for 5 weeks?

[How much for 1 week? how much for 4 weeks or a month? how much for 5 weeks?]

37. If 7 horses eat 14 bushels of oats in 1 week, how many bushels would 15 horses eat in the same time?

38. If 3 horses eat 8 bushels of oats in 2 weeks, how long would it take them to eat 40 bushels?

[40 bushels is how many times 8 bushels?]

39. If 1 horse eats 1 bushel of oats in 3 days, in how many days would 4 horses eat 36 bushels?

Answer: If 1 horse eats 1 bushel in 3 days, 4 horses will eat 4 times as much, or 4 bushels. If 4 horses eat 4 bushels in 3 days, to eat 36 bushels will take them as many times 3 days as 4 bushels is contained in 36 bushels, or 9 times 3 days, which is 27 days.

40. If 2 men spend 12 dollars in 1 week, how many dollars would 3 men, at the same rate, spend in 5 weeks?

[1st. How much will 1 man spend in 1 week? 2d. How much will he spend in 5 weeks? 3d. How much will 3 men spend in 5 weeks?]

41. If a staff 3 feet long casts a shadow of 2 feet at 4 o'clock in the afternoon, what is the length of a pole that casts a shadow of 18 feet at the same time of day?

[As many times 3 feet as 2 goes in 18.]

42. If in 1 hour 47 gallons of water run into a cistern which will hold 108 gallons, and 38 gallons run out, how much remains in the cistern in an hour? and how many hours will it take to fill the cistern?

43. If 4 men can do a piece of work in 8 days, how many men would it take to do the same work in 4 days?

44. If 6 men can do a piece of work in 9 days, in how many days would 2 men do it?

[2 men is $\frac{1}{3}$ of 6 men. With $\frac{1}{3}$ as many men, how much longer will it take?]

45. If 8 men can do a piece of work in 5 days, in how many days would they do a piece of work 4 times as large?

46. If 7 men can do a piece of work in 3 days, how many men would it take to do a piece of work 5 times as large in the same time?

47. If 8 men can do a piece of work in 4 days, in how many days would 2 men do a piece of work one half as large?

[How many days will it take 8 men to do *half* the work? how much longer will it take 2 men?]

48. A man bought a cask of oil containing 63 gallons, 3 sevenths of which leaked out; and he sold the remainder for 36 dollars; how much per gallon did he sell it for?

49. If a pile of wood costs 48 dollars, what is 5 eighths of it worth?

50. A man bought 7 oranges for 6 cents apiece, and sold them all for 54 cents; how much did he gain by the bargain?

51. A man bought 8 yards of cloth for 56 dollars, and sold it again for 9 dollars a yard; how much did he gain by the bargain?

52. A man bought 8 barrels of flour for 40 dollars; for how much must he sell it per barrel to gain 16 dollars?

[How much must he sell the whole for in order to gain 16 dollars? how much is this per barrel?]

53. A man bought 5 firkins of butter at 7 dollars a firkin; for how much must he sell it per firkin to gain 10 dollars?

54. A man gave 35 cents for his breakfast, which was 5 eighths of what he gave for his dinner; what did he give for his dinner?

55. A ship's crew of 6 men have provision for 3 months; how many months would it last 1 man?

56. A ship's crew have provision sufficient to last 1 man 27 months; how long would it last 9 men?

57. A ship's crew have provision sufficient to last 3 men 10 months; how long would it last 5 men?

[How long will it last 1 man? 5 men?]

58. A man built 40 rods of wall in 18 hours; how

long would it have taken another man who can build 9 rods while the first builds 5? *Answer:* 10 hours.

59. A boy gave away 2 apples and 1 half, which was 1 fourth of all he had; how many had he?

60. A man gave away 3 dollars, which was 2 fifths of all the money he had; how much had he?

61. A man sold a cow for 35 dollars, which was 4 fifths of what she cost him; how much did he lose by the bargain?

Answer: He lost 1 fifth of the cost; if 35 dollars is 4 fifths, 1 fifth is 1 fourth of 35, or $\frac{35}{4}$ dollars, which is $8\frac{3}{4}$ dollars. Therefore he lost $\$8\frac{3}{4}$.

62. A man sold a piece of cloth for 37 dollars, which was 9 eighths of what it cost him; how much did he gain by the bargain?

63. There is a pole 3 fifths under water, and 7 feet out of the water; how long is the pole?

64. A man sold a piece of cloth for 47 dollars, by which bargain he lost 2 ninths of what the cloth cost him; how much did it cost him, and how much did he lose?

65. If a staff 5 feet long casts a shadow 4 feet long at 4 o'clock in the afternoon, what is the length of a pole that casts a shadow 67 feet long at the same time?

66. If in 1 hour 53 gallons of water run into a cistern which will hold 97 gallons, and 44 gallons run out, in what time will it be filled?

67. A man bought 9 oranges for 6 cents and 2 sevenths apiece, and sold them for 67 cents; what did he gain by the bargain?

68. A man bought 10 yards of broadcloth for 70 dollars; how must he sell it per yard in order to gain 14 dollars?

69. If, when the days are 12 hours long, a man performs a journey in 3 days, how many hours is he in performing it?

70. If a man performs a journey in 36 hours, how many days would he be in performing it when the days are nine hours long?

71. If, when the days are 11 hours long, a man can perform a journey in 5 days, in how many hours will he perform it? in how many days when the days are 9 hours long?

72. What number added to 2 fifths of 33 will make the number 17?

73. How many yards of cloth, 1 quarter of a yard wide, will line 10 yards, 3 quarters wide?

74. 8 yards of cloth, 1 quarter wide, are equal to how many yards, 4 quarters wide?

75. How many yards of cloth, 3 quarters wide, are equal to 7 yards, 5 quarters wide?

Answer: 7 yards, 5 quarters wide, is the same as 7 times 5, or 35 yards, only 1 quarter wide; 35 yards, 1 quarter wide, is the same as $\frac{35}{3}$, or $11\frac{2}{3}$ yards, 3 quarters wide.

76. How many yards of cloth, 6 quarters wide, are equal to 37 yards, 4 quarters wide?

77. If a piece of cloth, 5 quarters wide, be worth 37 dollars, what is a piece of the same length, 3 quarters wide, worth?

78. If cloth, 4 quarters wide, is worth 8 dollars a yard, what is 1 yard of the same kind of cloth, 5 quarters wide, worth?

79. A man sold 8 yards of cloth for $3\frac{1}{3}$ dollars a yard; what did it come to?

80. A man sold a horse for 76 dollars, which was $\frac{8}{9}$ of what it cost him; how much did it cost him?

81. A man sold $\frac{3}{5}$ of a gallon of molasses for 40 cents; what was that a gallon?

82. If it will take $1\frac{1}{4}$ yard of cloth to make a coat, how many yards will it take to make 7 coats?

83. If 1 horse consumes $3\frac{1}{4}$ bushels of oats in 2 days, how much would 2 horses consume in 5 days?

84. If, when the days are $9\frac{1}{2}$ hours long, a man performs a journey in 10 days, in how many days would he perform it when the days are 12 hours long?

85. A man sold 8 yards of cloth for $7\frac{3}{5}$ dollars a yard, and received 8 firkins of butter at $6\frac{3}{5}$ dollars a firkin; how much was then due him?

86. Two men are 38 miles apart, and are traveling towards each other, one at the rate of 3 miles an hour, the other at the rate of 2 miles; how much do they approach each other in an hour? how much in 2 hours? In how many hours will they meet? How far will each have traveled at the time of meeting?

B.

1. Twenty-five are how many times 3? 7? 2? 6? 9? 4? 8? 5? 10?
2. Thirty are how many times 10? 2? 3? 7? 9? 6? 5? 4? 8?
3. Thirty-three are how many times 6? 8? 7? 4? 9? 5? 10? 3?
4. Twenty-six are how many times 9? 4? 7? 3? 8? 5? 6? 10?
5. Thirty-five are how many times 5? 6? 3? 7? 9? 10? 4? 8?
6. Thirty-eight are how many times 8? 6? 3? 9? 5? 4? 7? 10?
7. Thirty-four are how many times 7? 3? 9? 10? 6? 8? 4? 5?
8. Thirty-six are how many times 8? 9? 4? 5? 3? 6? 7? 10?
9. Forty are how many times 8? 10? 6? 4? 3? 9? 5? 7?
10. For forty-seven cents, how many pounds of fish can be bought at 6 cents a pound? how many pounds at 8 cents? how many at 9 cents? how many at 3 cents? how many at 5 cents? how many at 4 cents? how many at 7 cents? how many at 10 cents a pound?
11. Forty-seven are how many times 6? 8? 9? 3? 5? 4? 7? 10?
12. Forty-three are how many times 9? 8? 7? 6? 4? 3? 5? 10?
13. Forty-five are how many times 10? 8? 3? 6? 4? 7? 5? 9?
14. Forty-nine are how many times 6? 10? 5? 9? 4? 8? 7?

15. Fifty-three are how many times 8? 5? 6? 4? 7? 9? 10?

16. Fifty-seven are how many times 9? 7? 10? 6? 5? 8? 4?

17. Fifty-five are how many times 6? 4? 8? 10? 9? 7? 5?

18. Forty-eight are how many times 7? 5? 9? 4? 6? 8? 10?

19. Fifty-four are how many times 5? 9? 6? 4? 7? 10? 8?

20. Forty-four are how many times 4? 6? 9? 7? 5? 8? 10?

21. Fifty-eight are how many times 7? 6? 8? 4? 9? 5? 10?

22. Forty-six are how many times 8? 10? 4? 6? 9? 7? 5?

23. Fifty are how many times 9? 5? 4? 10? 8? 6? 7?

24. Fifty-nine are how many times 4? 8? 7? 6? 10? 9? 5?

25. Sixty-four are how many times 7? 5? 8? 10? 6? 9?

26. Sixty-eight are how many times 6? 8? 9? 7? 10? 5?

27. Fifty-two are how many times 4? 6? 8? 10? 5? 7? 9?

28. Sixty-three are how many times 5? 4? 6? 10? 9? 7? 8?

29. Sixty-two are many times 4? 10? 9? 7? 8? 5? 6?

30. Seventy-three are how many times 10? 7? 8? 6? 5? 9?

31. Seventy-five are how many times 7? 8? 10? 5? 6? 9?

32. If you had sixty-seven dollars, how many barrels of flour could you buy at 5 dollars a barrel? how many at 7 dollars a barrel? how many at 6 dollars a barrel? how many at 8 dollars a barrel? how many at 10 dollars a barrel? how many at 9 dollars a barrel?

33. Sixty-seven are how many times 5? 7? 6? 8? 10? 9?

34. Seventy-four are how many times 10? 7? 8? 6? 5? 9?

35. Sixty are how many times 9? 10? 6? 4? 7? 5? 8?

36. Seventy-two are how many times 5? 7? 6? 8? 10? 9?

37. Sixty-five are how many times 5? 10? 8? 6? 7? 9?

38. Sixty-one are how many times 4? 5? 7? 6? 8? 10? 9?

39. Seventy-nine are how many times 10? 9? 8? 6? 5?

40. Seventy are how many times 9? 5? 6? 8? 7? 10?

41. Eighty-two are how many times 10? 7? 8? 6? 9?

42. Sixty-six are how many times 9? 5? 6? 7? 10? 8?

43. Eighty are how many times 10? 7? 6? 8? 9?

44. Sixty-nine are how many times 9? 5? 7? 10? 8? 6?

45. Eighty-one are how many times 10? 6? 8? 7? 9?

46. Seventy-six are how many times 9? 5? 10? 6? 7? 8?

47. Eighty-three are how many times 10? 6? 7? 9? 8?

48. Seventy-one are how many times 9? 5? 7? 6? 8? 10?

49. Eighty-four are how many times 10? 6? 8? 5? 7?

50. Seventy-seven are how many times 9? 7? 5? 10? 8? 6?

51. Eighty-five are how many times 10? 8? 7? 6? 9?

52. Ninety are how many times 9? 10? 6? 7? 8?

53. Eighty-six are how many times 10? 9? 6? 7? 8?

54. Ninety-four are how many times 9? 10? 8? 6? 7?

55. Eighty-seven are how many times 10? 9? 7? 6? 8?

56. Ninety-two are how many times 9? 6? 7? 8?

57. Eighty-eight are how many times 10? 9? 8? 6? 7?

58. Ninety-five are how many times 9? 10? 6? 8? 7?

59. Eighty-nine are how many times 10? 9? 6? 7? 8?

60. Ninety-eight are how many times 10? 9? 8? 6? 7?

61. One hundred are how many times 6? 10? 8? 9? 7?

62. Ninety-three are how many times 10? 6? 9? 8? 7?

63. Ninety-nine are how many times 7? 10? 8? 6? 9?

64. Ninety-six are how many times 9? 8? 7? 6?

65. Ninety-seven are how many times 10? 9? 6? 7? 8?

66. At 8 shillings for 4 lbs., what would 10 lbs. cost?

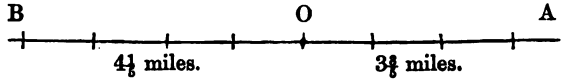
67. If a man consumes 1 lb. and 3 oz. of meat in a day, how much would he consume in a week?

68. If a man spends $2\frac{3}{5}$ dollars in a day, how much would he spend in a week?

69. If a man travels $3\frac{3}{4}$ miles in an hour, how far would he travel in 3 hours? how far in 7 hours? how far in 12 hours?

70. If 2 men, A and B, start from the same place, O, and travel in opposite directions, A at the rate of $3\frac{3}{5}$ miles in an hour, and B at the rate of $4\frac{1}{5}$ miles, how far apart will they be at the end of 1 hour? how far at the end of 2 hours? how far at the end of 3 hours? how far at the end of 7 hours?

This figure shows the relative positions of the two men A and B at the end of 1 hour:



71. Two men start from the same place, and travel the same way; one at the rate of $4\frac{1}{7}$ miles in an hour, the other at the rate of $4\frac{4}{7}$ miles in an hour; how far apart will they be at the end of 1 hour? how far in 2 hours? how far in 5 hours? how far in 10 hours? how far in 3 days, if they travel 10 hours in a day?

72. How many yards of cloth, at $5 a yard, must be given for 8 barrels of flour at $7 per barrel?

73. What part of a month is 1 week? 2 weeks? 3 weeks?

74. What part of a year is 1 month? 2 months? 3 months? 4 months? 5 months? 6 months? 7 months? 8 months? 9 months? 10 months? 11 months?

75. What part of 1 month is 1 day? 3 days? 7 days? 15 days? 18 days? 20 days? 24 days? 27 days?

76. If 5 bushels of oats will keep 7 horses a week, how many bushels will it take to keep 12 horses the same time?

77. If you give 7 men $2\frac{1}{8}$ bushels of corn apiece, how many bushels will it take?

78. A man who had failed in trade was able to pay his creditors only 40 cents on a dollar; how much would he pay on 2 dollars? how much on 3 dollars? how much on 7 dollars? how much on 10 dollars?

79. A man who has failed in trade is able to pay only 9 shillings on a pound; how much would he pay on a debt of 2 pounds? how much on 3 pounds? how much on 12 pounds?

80. A man who has failed in trade is able to pay only 62 cents on a dollar; how much would he pay on a debt of 7 dollars?

81. If 6 dollars' worth of provisions will serve 3 men 5 days, how many days will it serve 1 man? how many days will it serve 2 men? how long will it serve 8 men?

82. If 10 dollars' worth of provisions will serve 7 men 4 days, how many days will it serve 1 man? 9 men?

83. If 12 dollars' worth of provisions will serve 8 men 3 days, how many men would it serve 1 day? how many 2 days? how many 8 days?

84. If 11 dollars' worth of provisions will serve 6 men 8 days, how many men will it serve 4 days?

85. If 8 dollars' worth of provisions will serve 7 men 5 days, how many days would 16 dollars' worth of provisions last 4 men?

86. If 1 peck of wheat affords 11 eight-cent loaves, how many ten-cent loaves will it afford?

87. A man having hired some men and some boys, agreed to give each man \$3, and each boy \$2; how much would it take to pay a man and a boy? how much 2 men and 2 boys? how much 7 men and 7 boys?

88. A man paid \$18 to his laborers: he gave to every man \$2, and to every boy \$1; the number of men and boys was equal; how many were there of each?

89. A gentleman paid 50 shillings to his laborers; he gave to every man 8 pence, and to every boy 4 pence; the number of men and boys was equal; how many were there of each?

Solution: Every man has a boy to help him; each man and boy receive together 8 pence + 4 pence, or 12 pence, which is 1 shilling. 50 shillings, then, will pay 50 men and 50 boys.

90. Two men bought a bushel of corn: one gave 1 shilling, the other 2 shillings; what part of the whole did each pay? What part of the corn must each have?

[The whole bushel cost 3 shillings, and one man paid $\frac{2}{3}$ of this, and the other $\frac{1}{3}$.]

91. Two men bought a barrel of flour for 8 dollars: one gave 3 dollars, the other 5 dollars; what part did each pay? and what part must each have?

92. Three men, A, B, and C, hired a garden: A paid 6 dollars, B 5 dollars, and C 9 dollars; how much did they all pay? What part of the whole did each pay? They sold the produce for 40 dollars; what part of it did each have? How much did each gain?

[The rent of the garden was \$6 + \$5 + \$9, or \$20. So that A should have $\frac{6}{20}$ of the profits, B $\frac{5}{20}$, and C $\frac{9}{20}$.]

93. Two men hired a pasture for \$58; one put in 7 horses, and the other 3 horses; what had each to pay?

94. Three men, Smith, Jones, and Robinson, commenced trade together; they put in money in the following proportion: Smith \$3 as often as Jones put in \$4, and as often as Robinson put in \$5; they gained \$87; what was each man's share of the gain?

[Smith put in $\frac{4}{19}$ of the whole capital, and should have $\frac{8}{19}$ of the profits. Jones put in $\frac{4}{19}$ of the capital, and Robinson $\frac{5}{19}$.]

95. Two men hired a pasture for 32 dollars; the first put in 3 sheep for 4 months, the second put in 4 sheep for 5 months; how much ought each to pay?

[Three sheep for 4 months is the same as 12 sheep for 1 month; 4 sheep for 5 months the same as 20 sheep for 1 month. This question is therefore the same as if 1 man put in 12 sheep, and the other 20 sheep.]

96. Two men, A and B, traded in company: A put in \$1 for 4 months, and B \$2 for 3 months, and they gained 90 cents; how many cents must each have?

97. Three men, A, B, and C, traded in company, and put in money in the following proportions: A put in 4 dollars as often as B put in 3, and as often as C put in 2; A's money was in 2 months, B's 3 months, and C's 4 months, and they gained 100 dollars; what was each one's share?

[\$4 for 2 months is the same as \$8 for 1 month; \$3 for 3 months is the same as \$9 for 1 month; and \$2 for 4 months is the same as \$8 for 1 month.]

98. Two men, A and B, traded in company: A put in 2 dollars as often as B put in 3; A's money was employed 7 months, and B's money 5 months: they gained 58 dollars; what was each man's share of the gain?

99. Three men, A, B, and C, traded in company, and put in money in the following proportions: A put in 2 dollars as often as B put in 4, and as often as C put in 6. B's money was in twice as long as C's, and A's two times as long as B's; they gained 88 dollars; what was each one's share of the gain?

[A's money was in 4 times as long as C's. It is the same as if A had put in \$8, B \$8, and C \$6, all for the same time. A must have $\frac{8}{22}$, B $\frac{8}{22}$, and C $\frac{6}{22}$ of \$88.

C.

[Interest is a reward or premium allowed by a debtor to a creditor for the use of money. The usual interest for 1 year, and that which is established by law in twenty-three of the States of this Union, is 6 cents on a dollar, 6 dollars on a hundred dollars, or, in fine, $\frac{6}{100}$ of the sum, whatever be the denomination. It is called 6 per cent., that is, 6 on the hundred, because it is always reckoned by the hundred. So 3 per cent., 4 per cent., etc., signify $\frac{3}{100}$, $\frac{4}{100}$, etc., or so much on the hundred. The teacher can vary the examples to illustrate the different rates per cent.]

1. The interest of 1 dollar being 6 cents for 1 year, what is the interest of 7 dollars for the same time? What is the interest of \$10? of \$15? of \$20? of \$30? of \$50? of \$75? of \$100? of \$118?

2. If the interest of 1 dollar is 6 cents for 1 year, what would it be for 2 years? What would be the interest of \$8 for two years? of \$17? of \$43?

3. If the interest of 100 dollars is 6 dollars for a year, what would be the interest of 50 dollars for the same time? of 2 hundred? of 3 hundred? of 4 hundred? of 1 hundred and 50? of 2 hundred and 50?

4. If the interest of 100 dollars is 6 dollars for 1 year, what would it be for 6 months? for 3 months? for 4 months? for 8 months? for 9 months? for 1 month? for 2 months? for 5 months? for 7 months? for 10 months? for 11 months?

5. What is the interest of 100 and 32 dollars for 2 years, at 6 per cent.?

6. What is the interest of 300 dollars for 1 year and 6 months, at 6 per cent.?

7. What is the interest of 1 dollar for 6 months, at 6 per cent.? what for 2 months? what for 1 month? what for 3 months? 4 months? 5 months? 7 months? 9 months? 11 months?

8. What is the interest of 57 dollars for 1 year and 7 months, at 6 per cent.?

Solution: The interest of 50 dollars for 1 year and 6 months is 4 dollars and 50 cents, and for 1 month it is

25 cents. The interest of 7 dollars for 19 months (the interest of a dollar is $\frac{1}{2}$ of a cent a month) is $66\frac{1}{2}$ cents. The whole amounts to 5 dollars and $41\frac{1}{2}$ cents.

9. What is the interest of 200 and 67 dollars for 1 year and 4 months, at 6 per cent.?

Solution: The interest of 200 dollars for $1\frac{1}{3}$ years is 16 dollars. The interest of 67 dollars is 67 cents for every 2 months; for 16 months it will be 8 times 67 cents, which are 5 dollars and 36 cents. The whole interest is 21 dollars and 36 cents.

10. If the interest for 1 year is 6 per cent., what would be the per cent. for 2 years? for 3 years? for 6 months? for 2 months? for 1 month? for 4 months? for 5 months? for 7 months? for 8 months? for 9 months?

11. If the interest for 2 months or 60 days* is 1 per cent., what would be the per cent. for 20 days? what for 40 days? what for 15 days? what for 45 days? what for 12 days? what for 10 days? what for 5 days?

12. What is the interest of 100 and 37 dollars for 20 days?

Solution: The interest of 100 dollars for 60 days is 1 dollar, and for 20 days is $\frac{1}{3}$ of a dollar, or $33\frac{1}{3}$ cents. The interest of 37 dollars for 60 days is 37 cents, and for 20 days $\frac{1}{3}$ of 37 cents, or $12\frac{1}{3}$ cents. The whole interest is $33\frac{1}{3}+12\frac{1}{3}$ cents, or $45\frac{2}{3}$ cents.

13. A can do a piece of work in 2 days; how much of it can he do in 1 day?

14. B can do a piece of work in 4 days; how much of it can he do in 1 day?

15. If A can do $\frac{1}{2}$ of a piece of work in 1 day, and B can do $\frac{1}{4}$ of it in 1 day, how much would both do in a day? How long would it take them both together to do the whole?

Solution: They would both together do $\frac{3}{4}$ of the work in 1 day, and it would take them $\frac{1}{3}$ of a day to do the other $\frac{1}{4}$. *Answer:* $1\frac{1}{3}$ days.

16. If 1 man can do a piece of work in 2 days, and another in 3 days, how much of it would each do in a

* In computing interest, a month is usually regarded as 30 days.

day? How much would both together do? how long would it take them both to do the whole?

17. A cistern has 2 pipes; the first will fill it in 3 hours, the second in 6 hours; how much of it would each fill in an hour? How much would both together fill? How long would it take them both to fill it?

18. A man and his wife found by experience that, when they were both together, a bushel of meal would last them only 2 weeks; but when the man was gone it would last his wife 5 weeks; how much of it did both together consume in 1 week? What part did the woman alone consume in 1 week? What part did the man alone consume in 1 week? How long would it last the man alone?

Solution: They both together consume $\frac{1}{2}$ of a bushel in a week, but the woman alone consumes only $\frac{1}{5}$ of a bushel in a week. That is, they both together consume $\frac{5}{10}$ in a week, but the woman alone only $\frac{2}{10}$; consequently, the man alone would consume $\frac{3}{10}$, and a bushel would last him $3\frac{1}{3}$ weeks.

19. If 1 man could build a piece of wall in 5 days, and another man could do it in 7 days; how much of it could each do in 1 day? How many days would it take them both to do it?

20. A cistern has 3 pipes; the first would fill it in 3 hours; the second in 6 hours; the third in 4 hours; what part of the whole would each fill in 1 hour? and how long would it take them all to fill it, if they were all running at once?

21. A and B together can build a boat in 8 days, and with the assistance of C they can do it in 5 days; how much of it can A and B build in 1 day? How much of it can A, B, and C build in 1 day? How much of it can C alone build in 1 day? How long would it take C to build it alone?

Solution: A and B can build $\frac{1}{8}$ of it in 1 day; A, B, and C can build $\frac{1}{5}$ of it in 1 day; the difference between $\frac{1}{5}$ and $\frac{1}{8}$ is $\frac{3}{40}$; therefore C can build $\frac{3}{40}$ of it in 1 day, and it would take him $13\frac{1}{3}$ days to build it alone.

22. Suppose I should line 8 yards of broadcloth, which is $1\frac{1}{2}$ yards wide, with alpaca which is $\frac{3}{4}$ of a yard wide; how many yards of the alpaca will line 1 yard of the broadcloth? How many yards will line the whole?

23. If 7 yards of cloth cost 13 dollars, what will 10 yards cost?

24. If the wages of 25 weeks come to 75 dollars, what will be the wages of 7 weeks?

25. If 8 tons of hay will keep 7 horses three months, how much will keep 12 horses the same time?

26. If a staff 4 feet long casts a shadow 6 feet long, what is the length of a pole that casts a shadow of 58 feet at the same time of day?

[As many times 4 as 6 is contained in 58.]

27. If a stick 8 feet long casts a shadow 2 feet in length, what is the height of a tree which casts a shadow of 42 feet at the same time of day?

28. At 6 dollars per week, how many months' board can I have for 100 dollars?

29. A ship has sailed 24 miles in 4 hours; how long will it take her to sail 150 miles at the same rate?

30. 30 men can perform a piece of work in 20 days; how many men will it take to perform the same work in 8 days?

31. 17 men can perform a piece of work in 25 days; in how many days would 5 men perform the same work?

32. A hare has 76 rods the start of a greyhound, but the greyhound runs 15 rods to 10 of the hare; how many rods must the greyhound run to overtake the hare?

[The greyhound must *gain* 76 rods; he has to run 15 rods in order to *gain* 5; therefore, in order to gain 76 rods, he must run as many times 15 rods as 5 is contained in 76.]

33. A garrison has provisions for 8 months, at the rate of 15 ounces per day; how much must be allowed per day, in order that the provisions may last 11 months?

[Find how much they might eat in a day, in order to make

it last 1 month, and then it will be easy to find how much they may eat in a day, to make it last 11 months.]

34. If 8 men can build a wall 15 rods in length in 10 days, how many men will it take to build a wall 45 rods in length in 5 days?

35. If 4 bushels of wheat afford 60 ten-cent loaves, how many eight-cent loaves may be obtained from it?

36. Said Harry to Dick, "My purse and money together are worth $16, but the money is worth 7 times as much as the purse." How much money was there in the purse? and what was the value of the purse?

[The money is 7 parts of the whole, and the purse one part; consequently the money is $\frac{7}{8}$ and the purse $\frac{1}{8}$ of 16.]

37. A man being asked the price of his horse, answered, that his horse and saddle together were worth 100 dollars, but the horse was worth 9 times as much as the saddle; what was each worth?

38. A man having a horse, a cow, and a sheep, was asked what was the value of each. He answered, that the cow was worth twice as much as the sheep, and the horse 3 times as much as the sheep, and that all together were worth 120 dollars; what was the value of each?

39. A man bought an apple, an orange, and a melon for 21 cents; for the orange he gave twice as much as for the apple, and for the melon he gave twice as much as for the orange; how much did he give for each?

[He gave one part for the apple, 2 parts for the orange, and 4 parts for the melon. These make 7 parts. The apple 3 cents, the orange 6 cents, and the melon 12 cents.]

40. If 80 dollars' worth of provisions will serve 20 men 24 days, how many days will 100 dollars' worth of provisions serve 30 men?

41. There is a pole $\frac{1}{3}$ and $\frac{1}{5}$ under water, and 10 feet out; how long is the pole?

42. In an orchard of fruit trees, $\frac{1}{2}$ of them bear apples, $\frac{1}{4}$ of them bear plums, $\frac{1}{8}$ of them pears, 7 of them peaches, and 3 of them cherries; how many trees are there in the whole, and how many of each sort?

43. A farmer being asked how many sheep he had,

answered, that he had them in 4 pastures; in the first he had $\frac{1}{3}$ of his flock; in the second $\frac{1}{4}$; in the third $\frac{1}{6}$; and in the fourth 15; how many sheep had he?

44. A man driving his geese to market, was met by another, who said, "Good-morrow, master, with your hundred geese;" says he, "I have not a hundred; but if I had half as many more as I now have, and two geese and a half, I should have a hundred." How many had he?

[If to a number half of itself be added, the sum is $\frac{3}{2}$ of that number; hence subtract $2\frac{1}{2}$ from 100, and the remainder is $\frac{3}{2}$ of the number of geese that he had.]

45. What number is that, to which if its half be added the sum will be 60?

46. What number is that, to which if its third be added the sum will be 48?

47. What number is that, to which if its fifth be added the sum will be 54?

48. What number is that, to which if its half and its third be added the sum will be 55?

49. A man being asked his age, answered, that if its half and its third were added to it, the sum would be 77; what was his age?

[This must be reduced to 6ths. 1 half is $\frac{3}{6}$, and $\frac{1}{3}$ is $\frac{2}{6}$, and the number itself is $\frac{6}{6}$. If, therefore, to the whole number its half and its third be added, the sum will be $\frac{11}{6}$; hence, 77 is $\frac{11}{6}$ of the number.]

50. What number is that which, being increased by its half, its fourth, and eighteen more, will be doubled?

[$\frac{1}{2}$ is $\frac{2}{4}$; therefore, if to a number $\frac{1}{2}$ and $\frac{1}{4}$ of itself be added, the whole number will be $\frac{7}{4}$; but when 18 more is added to $\frac{7}{4}$, the first number is doubled, and the result is $\frac{8}{4}$; therefore 18 is $\frac{1}{4}$ of the number.]

51. A boy being asked his age, answered, that if $\frac{1}{2}$ and $\frac{1}{4}$ of his age, and 20 more, were added to his age, the sum would be 3 times his age; what was his age?

52. A man being asked how many sheep he had, answered, that if he had as many more, $\frac{1}{2}$ as many more, and $2\frac{1}{2}$ sheep, he should have 100; how many had he?

APPENDIX.

WARREN COLBURN.

[*A brief sketch of his life, principally taken from Barnard's "Journal of Education," vol. ii., 1856, to which the reader is referred for more detailed information.*]

WARREN COLBURN was born March 1, 1793, in the part of Dedham, Mass., called Pond Plain. His father was a farmer and the son was early required to do a boy's work on the farm. Warren showed, when quite young, a taste for arithmetic, in the study of which he was much encouraged by his father.

From 1810 to 1815 he worked on machinery at Pawtucket, R. I., and at Plymouth, Mass. What little schooling he had thus far received had been at the district school, and principally in the winter.

In the summer of 1815, at the age of twenty-two and a half years, he began to fit for Harvard College, which he entered in 1817 and from which he was graduated in 1820. He was considered by his classmates to be by far the best mathematician in the class. While in college he taught school in the winter months in Boston, in Leominster, and in Canton.

From 1820 to 1823 he successfully taught a private school in Boston. In 1821 was published his "First Lessons in Intellectual Arithmetic," in regard to which he is said to have remarked that the pupils who were under his tuition made the book for him, and that he had only given attention to the questions they asked.

We quote the following paragraph from Barnard's "Journal of Education" referred to above, page 302.

"Thus the 'First Lessons' worked its way gradually to notice and favor, — a book which has enjoyed a more enviable success than any other school-book ever published in this country, and the merits of which are now universally acknowledged to be equal to its success. It has been said to be 'the only faultless school-book that we have.'

It has certainly wrought a great change in the manner of teaching arithmetic.

"Its system is received wherever the book is known. It has no competitors, except in the profits of sale, in the shape of imitations; and that these have been numerous is altogether to its credit. Such a man as George B. Emerson, after twelve years' constant use of it, long ago pronounced it the most valuable school-book that has made its appearance in this country. And Thomas Sherwin, Esq., of the Boston High School, calls it not only the best in this country but the best in the world. Its use is believed to be nearly commensurate with that of the English language, and it has been translated into other tongues. It has been stated that fifty thousand copies of Colburn's 'First Lessons' are annually used in Great Britain: and its sale in this country is about one hundred thousand per annum. About two millions of copies have been sold since its first publication in this country."

This was written in 1856; since then the "First Lessons" has been so extensively used that there is to-day probably no book by an American author that is more widely known.

In 1823 he was appointed Superintendent of the Boston Manufacturing Company's Works at Waltham, Mass., — one of the first establishments in the country for the manufacture of cotton goods. The mechanical genius which he here displayed in applying power to machinery, together with his great administrative ability, led to his appointment in 1824 as Superintendent of the Lowell Merrimac Manufacturing Company at Lowell, Mass.

For several years he delivered gratuitous lectures on the Natural History of Animals, Light, Electricity, the Seasons, Hydraulics, Eclipses, etc.; his knowledge of machinery enabled him admirably to illustrate these lectures by models of his own construction; and his successful experiments and simple teaching added much to the practical knowledge of his operatives.

He proposed to occupy the space between the common schools and the college halls by carrying, so far as might be found practicable, the design of the Rumford Lectures of Harvard into the community of the actual workers of common life.

His "Algebra upon the Inductive Method of Instruction" appeared in 1825, and his "Sequel to Intellectual Arithmetic" in 1826; he regarded the Sequel as a book of more merit and importance than the "First Lessons."

He also published a series of selections from Miss Edgeworth's stories, in a suitable form for reading exercises for the younger classes of the Lowell schools, in the use of which the teachers were carefully instructed.

In May, 1827, he was elected a fellow of the American Academy of Sciences. For several years he was a member of the Examining Committee for Mathematics at Harvard College.

He was a member of the superintending school committee of Lowell; and so busy were he and his co-workers, that they were repeatedly obliged to hold their meetings at six o'clock in the morning.

Warren Colburn seems to have been ardently admired — almost revered — by the teachers who have understandingly used his "First Lessons." We quote below a few sentences from some of his admirers: —

"I regard Mr. Colburn as the great benefactor of his age, with respect to the proper development of the mathematical powers. Pestalozzi, indeed, first conceived the plan; but Mr. Colburn realized the plan, popularized it, and rendered it capable of being applied by the humblest mediocrity." — MR. THOMAS SHERWIN, *formerly Principal of the Boston High School.*

"'Colburn's First Lessons' have undoubtedly done more to improve the methods of teaching, not only of numbers, but of language and other branches, than any other school-book published during the last half century.

"The method of 'Practice before Theory' so earnestly contended for by Mr. Herbert Spencer, and all writers on education, was for the first time, in this country, embodied in the works of Colburn." — MR. B. F. TWEED, *recently a Supervisor of Boston Schools.*

"'Colburn's First Lessons" is regarded by all intelligent educators as the greatest educational work that has appeared in this country during the present century. Its influence has not been confined to the mathematical branches of instruction, it has extended to all; nor do I believe that there has been any essential improvement made upon Colburn in the numberless elementary works that have appeared." — MR. ELBRIDGE SMITH, *Master of the Dorchester High School.*

There has been no teacher to whom the educators of the past fifty years have been so ready to accord hearty and enthusiastic praise.

In personal appearance Mr. Colburn was decidedly pleasing. His height was five feet ten, and his figure was well proportioned. His face was one not to be forgotten; it indicated sweetness of disposition, benevolence, intelligence, and refinement. His mental operations were not rapid, and it was only by great patience and long-continued thought that he achieved his objects. He was not fluent in conversation, his hesitancy of speech, however, was not so great when with friends as with strangers. The tendency of his mind was toward the practical in knowledge; his study was to simplify science, and to make it accessible to common minds. He died September 13, 1833, at the age of forty years.

WARREN COLBURN'S FIRST LESSONS.

PREFACE TO THE FIRST EDITION (1821).

As soon as a child begins to use his senses, nature continually presents to his eyes a variety of objects; and one of the first properties which he discovers is the relation of number. He intuitively fixes upon *unity* as a measure, and from this he forms the idea of more and less; which is the idea of quantity.

The names of a few of the first numbers are usually learned very early; and children frequently learn to count as far as a hundred before they learn their letters.

As soon as children have the idea of more and less, and the names of a few of the first numbers, they are able to make small calculations. And this we see them do every day about their playthings, and about the little affairs which they are called upon to attend to. The idea of more and less implies addition; hence they will often perform these operations without any previous instruction. If, for example, one child has three apples, and another five, they will readily tell how many they both have; and how many one has more than the other. If a child be requested to bring three apples for each person in the room, he will calculate very readily how many to bring, if the number does not exceed those he has learnt. Again, if a child be requested to divide a number of apples among a certain number of persons, he will contrive a way to do it, and will tell how many each must have. The method which children take to do these things, though always correct, is not always the most expeditious.

The fondness which children usually manifest for these exercises, and the facility with which they perform them, seem to indicate that the science of numbers, to a certain extent, should be among the first lessons taught to them.

To succeed in this, however, it is necessary rather to furnish occasions for them to exercise their own skill in performing examples, than to give them rules. They should be allowed

to pursue their own method first, and then they should be made to observe and explain it; and if it was not the best, some improvement should be suggested. By following this mode, and making the examples gradually increase in difficulty, experience proves, that at an early age children may be taught a great variety of the most useful combinations of numbers.

Few exercises strengthen and mature the mind so much as arithmetical calculations, if the examples are made sufficiently simple to be understood by the pupil; because a regular though simple process of reasoning is requisite to perform them, and the results are attended with certainty.

The idea of number is first acquired by observing sensible objects. Having observed that this quality is common to all things with which we are acquainted, we obtain an abstract idea of number. We first make calculations about sensible objects; and we soon observe that the same calculations will apply to things very dissimilar; and finally, that they may be made without reference to any particular things. Hence, from particulars, we establish general principles, which serve as the basis of our reasonings, and enable us to proceed, step by step, from the most simple to the more complex operations. It appears, therefore, that mathematical reasoning proceeds as much upon the principle of analytic induction, as that of any other science.

From the above observations, and from his own experience, the author has been induced to publish this treatise; in which he has pursued the following plan, which seemed to him the most agreeable to the natural progress of the mind.

GENERAL VIEW OF THE PLAN.

EVERY combination commences with practical examples. Care has been taken to select such as will aptly illustrate the combination, and assist the imagination of the pupil in performing it. In most instances, immediately after the practical, abstract examples are placed, containing the same numbers and the same operations, that the pupil may the more easily observe the connection. The instructor should be careful to make the pupil observe the connection. After these are a few abstract examples, and then practical questions again.

The examples are to be performed in the mind, or by means of sensible objects, such as beans, nuts, etc. The

pupil should first perform the examples in his own way, and then be made to observe and tell how he did them, and why he did them so.* †

When the pupil is made familiar with all the principles contained in this book, he will be able to perform all examples in which the numbers are so small that the operations may be performed in the mind. Afterwards he has only to learn the application of figures to these operations, and his knowledge of arithmetic will be complete.

The following are some of the principal difficulties which a child has to encounter in learning arithmetic, in the usual way, and which are seldom overcome. First, the examples are so large that the pupil can form no conception of the numbers themselves; therefore it is impossible for him to comprehend the reasoning upon them. Secondly, the first examples are usually abstract numbers. This increases the difficulty very much; for even if the numbers were so small that the pupil could comprehend them, he would discover but very little connection between them and practical examples. Abstract numbers, and the operations upon them, must be learned from practical examples; there is no such thing as deriving practical examples from those which are abstract, unless the abstract have been first derived from those which are practical. Thirdly, the numbers are expressed by fig-

* It is remarkable that a child, although he is able to perform a variety of examples which involve addition, subtraction, multiplication, and division, recognizes no operation but addition. Indeed, if we analyze these operations when we perform them in our minds, we shall find that they all reduce themselves to addition. They are only different ways of applying the same principle. And it is only when we use an artificial method of performing them, that they take a different form.

If the following questions were proposed to a child, his answers would be, in substance, like those annexed to the questions. How much is five less than eight? Ans.: Three. Why? Because five and three are eight. What is the difference between five and eight? Ans.: Three. Why? Because five and three are eight. If you divide eight into two parts, such that one of the parts may be five, what will the other be? Ans.: Three. Why? Because five and three are eight.

How much must you give for four apples at two cents apiece? Ans.: Eight cents. Why? Because two and two are four, and two are six, and two are eight.

How many apples, at two cents apiece, can you buy for eight cents? Ans.: Four. Why? Because two and two are four, and two are six, and two are eight.

We shall be further convinced of this, if we observe that the same table serves for addition and subtraction; and another table, which is formed by *addition*, serves both for multiplication and division.

This remark shows the necessity of making the pupil attend to his manner of performing the examples and of explaining to him the difference between them.

† A few remarks, which relate only to the arrangement of sections in the *original* edition, are here omitted.

ures, which, if they were used only as a contracted way of writing numbers, would be much more difficult to be understood at first than the numbers written at length in words. But they are not used merely as words, they require operations peculiar to themselves. They are, in fact, a new language, which the pupil has to learn. The pupil, therefore, when he commences arithmetic, is presented with a set of *abstract* numbers, written with *figures*, and so large that he has not the least conception of them even when expressed in *words*. From these he is expected to learn what the figures signify, and what is meant by addition, subtraction, multiplication, and division; and at the same time how to perform these operations with figures. The consequence is, that he learns only one of all these things, and that is, how to perform these operations on figures. He can perhaps translate the figures into words, but this is useless, since he does not understand the words themselves. Of the effect produced by the four fundamental operations he has not the least conception.

After the abstract examples, a few practical examples are usually given, but these again are so large that the pupil cannot reason upon them, and consequently he could not tell whether he must add, subtract, multiply, or divide, even if he had an adequate idea of what these operations are.

The common method, therefore, entirely reverses the natural process; for the pupil is expected to learn general principles before he has obtained the particular ideas of which they are composed.

The usual mode of proceeding is as follows: The pupil learns a rule, which, to the man that made it, was a general principle; but with respect to *him*, and oftentimes to the instructor himself, it is so far from it, that it hardly deserves to be called even a mechanical principle. He performs the examples, and makes the answers agree with those in the book, and so presumes they are right. He is soon able to do this with considerable facility, and is then supposed to be master of the rule. He is next to apply his rule to practical examples; but if he did not find the examples under the rule, he would never so much as mistrust they belonged to it. But finding them there, he applies his rule to them, and obtains the answers, which are in the book, and this satisfies him that they are right. In this manner he proceeds from rule to rule through the book.

When an example is proposed to him, which is not in the book, his sagacity is exercised, not in discovering the opera-

tions necessary to solve it, but in comparing it with the examples which he has performed before, and endeavoring to discover some analogy between it and them, either in the sound, or in something else. If he is fortunate enough to discover any such analogy, he finds what rule to apply; and if he has not been deceived in tracing the analogy, he will probably solve the question. His knowledge of the principles of his rules is so imperfect, that he would never discover to which of them the example belongs, if he did not trace it, by some analogy, to the examples which he had found under it.

These observations do not apply equally to all; for some will find the right course themselves, whatever obstacles be thrown in their way. But they apply to the greater part; and it is probable that there are very few who have not experienced more or less inconvenience from this mode of proceeding. Almost all who have ever fully understood arithmetic have been obliged to learn it over again in their own way. And it is not too bold an assertion to say, that no man ever actually learned mathematics in any other method than by analytic induction; that is, by learning the principles by the examples he performs; and not by learning principles first, and then discovering by them how the examples are to be performed.

THE BOY WITHOUT A GENIUS.

Mr. Wiseman, the schoolmaster, at the end of his summer vacation, received a new scholar, with the following letter.

Sir, — This will be delivered to you by my son, Samuel, whom I beg leave to commit to your care, hoping that by your well-known skill and attention you will be able to make something of him; which, I am sorry to say, none of his masters have hitherto done. He is now eleven, and yet can do nothing but read his mother-tongue, and that but indifferently. We sent him at seven to a grammar school in our neighborhood; but his master soon found that his genius was not turned to learning languages. He was then put to writing, but he set about it so awkwardly that he made nothing of it. He was tried at accounts, but it appeared that he had no genius for that neither. He could do nothing in geography for want of memory. In short, if he has any genius at all, it does not yet show itself. But I trust to your experience in cases of this nature to discover what he is fit for, and to instruct him accordingly. I beg to be favored shortly with your opinion about him, and remain, sir,

Your most obedient servant,

Humphrey Acres.

When Mr. Wiseman had read this letter, he shook his head, and said to his assistant, A pretty subject they have sent us here! a lad that

has a great genius for nothing at all. But perhaps my friend Mr. Acres expects that a boy should show a genius for a thing before he knows anything about it, — no uncommon error! Let us see, however, what the youth looks like. I suppose he is a human creature at least.

Master Samuel Acres was now called in. He came, hanging down his head, and looking as if he was going to be flogged.

Come hither, my dear! said Mr. Wiseman. Stand by me, and do not be afraid. Nobody will hurt you. How old are you?

Eleven last May, sir.

A well-grown boy of your age, indeed. You love play, I dare say?

Yes, sir.

What, are you a good hand at marbles?

Pretty good, sir.

And can spin a top and drive a hoop, I suppose?

Yes, sir.

Then you have the full use of your hands and fingers?

Yes, sir.

Can you write, Samuel?

I learned it a little, sir, but I left it off again.

And why so?

Because I could not make the letters.

No! why, how do you think other boys do? Have they more fingers than you?

No, sir.

Are you not able to hold a pen as well as a marble?

Samuel was silent.

Let me look at your hand.

I see nothing here to hinder you from writing as well as any boy in the school. You can read, I suppose?

Yes, sir.

Tell me, then, what is written over the school-room door?

Samuel, with some hesitation, read WHATEVER MAN HAS DONE MAN MAY DO.

Pray, how did you learn to read? Was it not with taking pains?

Yes, sir.

Well, — taking more pains will enable you to read better. Do you know anything of the Latin grammar?

No, sir.

Have you never learned it?

I tried, sir, but I could not get it by heart.

Why, you can say some things by heart. I dare say you can tell me the names of the days of the week in their order.

Yes, sir, I know them.

And the months in the year, perhaps?

Yes, sir.

And you could probably repeat the names of your brothers and sisters, and all your father's servants, and half the people in the village besides?

I believe I could, sir.

Well, — and is *hic, hæc, hoc*, more difficult to remember than these?

Samuel was silent.

Have you learned anything of accounts?

I went into addition, sir, but I did not go on with it.

Why so?
I could not do it, sir.
How many marbles can you buy for a penny?
Twelve new ones, sir.
And how many for a half-penny?
Six.
And how many for two-pence?
Twenty-four.
If you were to have a penny a day, what would that make in a week?
Seven-pence.
But if you paid two-pence out of that, what would you have left.
Samuel studied a while, and then said, five-pence.

Right. Why, here you have been practicing the four great rules of arithmetic, — addition, subtraction, multiplication, and division. Learning accounts is no more than this. Well, Samuel, I see what you are fit for. I shall set you about nothing but what you are able to do; but observe, you *must* do it. We have no *I can't* here. Now, go among your school-fellows.

Samuel went away, glad that his examination was over, and with more confidence in his powers than he had felt before.

The next day he began business. A boy less than himself was called out to set him a copy of letters, and another was appointed to hear him in grammar. He read a few sentences in English, that he could perfectly understand, to the master himself. Thus by going on steadily and slowly, he made a sensible progress. He had already joined his letters, got all the declensions perfectly, and half the multiplication-table, when Mr. Wiseman thought it time to answer his father's letter; which he did as follows:

Sir, — I now think it right to give you some information concerning your son. You perhaps expected it sooner, but I always wish to avoid hasty judgments. You mentioned in your letter that it had not yet been discovered which way his genius pointed. If by *genius* you meant such a decided bent of mind to any one pursuit as will lead to excel with little or no labor or instruction, I must say that I have not met with such a quality in more than three or four boys in my life and your son is certainly not among the number. But if you mean only the *ability* to do some of those things which the greater part of mankind can do when properly taught, I can affirm that I find in him no peculiar deficiency. And whether you choose to bring him up to a trade or some practical profession, I see no reason to doubt that he may in time become sufficiently qualified for it. It is my favorite maxim, sir, that everything most valuable in this life may generally be acquired by taking pains for it. Your son has already lost much time in the fruitless expectation of finding out what he would take up of his own accord. Believe me, sir, few boys will take up anything of their own accord but a top or a marble. I will take care, while he is with me, that he loses no more time this way, but is employed about things that are fit for him, not doubting that we shall find him fit for them. I am, sir, yours, &c.,

Solon Wiseman.

Though the doctrine of this letter did not perfectly agree with Mr. Acres' notions, yet being convinced that Mr. Wiseman was more likely

to make something of his son than any of his former preceptors, he continued him at his school for some years, and had the satisfaction to find him going on in a steady course of gradual improvement. In due time a profession was chosen for him, which seemed to suit his temper and talents, but for which he had no *particular turn*, having never thought at all about it. He made a respectable figure in it, and went through the world with credit and usefulness, though *without a genius*.

Mrs. Barbauld.

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By allowing the class to study the lesson beforehand, not

only is much time lost, but the exercise is turned into a poor sort of mechanical process not much better than the common ciphering. Its mental character ceases almost entirely.

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